The Impact of Regulation on Innovation

By Philippe Aghion and Antonin Bergeaud and John Van Reenen*

We present a framework that can be used to assess the equilibrium impact of regulation on endogenous innovation with heterogeneous firms. We implement this model using French firm-level panel data where there is a sharp increase in the burden of labor regulations on companies with 50 or more employees. Consistent with the model's qualitative predictions, we find a sharp fall in the fraction of innovating firms just to the left of the regulatory threshold. Furthermore, we find a sharp reduction in the positive innovation response of firms to exogenous demand shocks just below the regulatory threshold. Using the structure of our model we quantitatively estimate parameters and find that the regulation reduces aggregate equilibrium innovation (and growth) by 5.7% which translates into a consumption equivalent welfare loss of at least 2.2%, approximately doubling the static losses in the existing literature.
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I. Introduction

There is considerable literature on the economic impacts of regulations, but relatively few studies on their impact on technological innovation. Most analyses focus on the static costs (and benefits) of regulation rather than on its dynamic effects. Yet these potential growth effects are likely to be much more important in the long run. Harberger triangles may be small, but rectangles can be very large. Many scholars have been concerned that slower growth in countries with heavy labor regulation, could be due to firms being reluctant to innovate due to the burden of red tape. For example, the slower growth of Southern European countries and parts of Latin America have often been blamed on onerous labor laws.[1]

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[1]See for example, Gust and Marquez (2004); Bentolila and Bertola (1990), Bassanini, Nunziata and Venn (2009), Schivardi and Schmitz (2020).

Identifying the innovation effects of labor regulation is challenging. The OECD, World Bank, IMF and other agencies have developed various indices of the importance of these regulations, based on examination of laws and surveys of managers. These indices are then often included in econometric models and are sometimes found to be significant. Unfortunately, these macro indices of labor law are correlated with many other unobservable factors that are hard to convincingly control for.[2] To address this issue, we exploit the fact that many regulations are size contingent and only apply when a firm gets sufficiently large. In particular, the burden of French labor legislation substantially increases when firms employ 50 or more workers. For example, such firms must create a works council with a minimum budget of 0.3% of total payroll, establish a health and safety committee, appoint a union representative and so on (see Appendix A for more institutional details). Several authors have found that these regulations have an important effect on the size of firms. Indeed, unlike the US firm size distribution, for example, in France, there is a clear bulge in the number of firms that are just below this regulatory threshold.[3]

Existing models that seek to rationalize these patterns have not usually considered how this regulation could affect innovation, as technology has been assumed exogenous. But when firms are choosing whether or not to invest in innovation, regulations are also likely to matter. Intuitively, firms may invest less in R&D as there is a very high cost of growing if the firm crosses the regulatory threshold. In the first part of the paper we formalize this intuition using a simple version of the Klette and Kortum (2004) model of growth and firm dynamics, with discrete time and two-period lived individuals (but potentially infinitely lived firms). Our model delivers a number of predictions regarding the shape of the equilibrium relationship between innovation and firm size and the overall firm size distribution. In particular we obtain the intuitive prediction that the regulatory threshold discourages innovation most strongly for firms just below the threshold, although it also discourages and shallows the innovation-size gradient for all firms larger than the threshold. This is because the growth benefits of innovation are lower due to the implicit regulatory tax.

We use the discontinuous increase in cost at the regulatory threshold to test the theory in two ways when taking it to our rich panel data on the population of French firms. First, we investigate non-parametrically how innovation changes with firm size. As expected there is a sharp fall in the fraction of innovative firms just to the left of the regulatory threshold, an "innovation valley" that is suggestive of a chilling effect of the regulation on the desire to grow. Moreover, there

[2]Furthermore, it may be that the more innovative countries are less likely to adopt such regulations (e.g. Saint-Paul, 2002).

[3]See Garicano, Lelarge and Van Reenen (2016); Gourio and Roys (2014); Ceci-Renaud and Chevalier (2011); and Smagghue (2020). Often, it is hard to see such discontinuities in the size distribution at regulation thresholds (e.g. Hsieh and Olken, 2014 and Amirapu and Gechter, 2020). A reason for the greater visibility in France is because the laws are more strictly enforced through large numbers of bureaucratic enforcers and strong trade unions.

is a flattening of the innovation-size relationship to the right of the threshold, consistent with a greater tax on growth. Although the cross-sectional evidence is suggestive, there could be many other reasons why firms are heterogeneous near the regulatory threshold, so we turn to a second and stronger test by exploiting the panel dimension of our data. Specifically, based on a wide class of models that predict that an increase in market size should have a positive effect on innovation (e.g. Acemoglu and Linn, 2004), we analyze the heterogeneous response of firms with different sizes to exogenous demand shocks. We use a shock based measure based on changes in growth in export product markets (disaggregated HS6 products by country destination) interacted with a firm's initial distribution of exports across these export markets (see Hummels et al., 2014 or Mayer, Melitz and Ottaviano, 2016). We first show that these positive market size shocks significantly raise innovative activity. We then examine the heterogeneity in firm responsiveness to these demand shocks depending on (lagged) firm size. We find a sharp reduction in firms' innovation response to the shock for firms with size just below the regulatory threshold. Consistent with intuition and our simple model, firms appear reluctant to take advantage of exogenous market growth through innovating when they will be subject to a wave of labor regulation.

Having established that the qualitative implications of the model are consistent with the data, we use the structure of our model (and empirical moments of the data) to quantitatively estimate the impact of the regulation on aggregate innovation and welfare. Our baseline estimates suggest that the regulation is equivalent to a tax on profit of about 2.6% that reduces aggregate innovation by around 5.7% (equivalent to cutting the annual growth rate from 1.7% to 1.6%) and reduces welfare by at least 2.2% in consumption equivalent terms. This is partly through misallocation from lowering entry and shifting the size distribution downwards, but the vast majority of this aggregate impact is through lower innovation per firm once they reach a certain size. This implies that the existing structural static analyses of the output loss have significantly underestimated the cost of the regulation.

A caveat to our welfare conclusions arises when we generalize our model to allow firms to invest in a mixture of radical and incremental innovation. We find that the regulation deters incremental R&D, but if a firm is going to innovate it will try to "swing for the fence" to avoid being only slightly to the right of the threshold. Measuring radical innovation by either future citations or a machine learning approach based on novelty in the patent text, we find that the negative effects of regulations are confined to incremental patents. Similarly, we find that regulation biases innovation towards automative labor-saving patents.

Related literature

Our paper relates to several strands of literature. More closely related to our analysis are papers that look at the effects of labor laws regulations on innovation. In Acharya, Baghai and Subramanian (2013*a*) higher firing costs reduce the risk that firms would use the threat of dismissal to hold their employees' innovative investments up. They find evidence in favor of this using macro time series variation on Employment Protection Law (EPL) for four OECD countries. Acharya, Baghai and Subramanian (2013*b*) also finds positive effects using staggered roll out of employment protection across US states.[4] Griffith and Macartney (2014) use multinational firms patenting activity across subsidiaries located in different countries with various levels of EPL.[5] Using this cross-sectional identification, they find that radical innovation was negatively affected by EPL, but incremental innovations were not.[6] By contrast, Alesina, Battisti and Zeira (2018) find that less regulated countries have larger high-tech sectors. All of these papers use macro (or at best, state-level) variation whereas we focus on cross-firm variation. Garcia-Vega, Kneller and Stiebale (2019) analyze a reform that relaxed a size contingent labor regulation in Spain and find an increase in innovation. Our empirical results are consistent with this, but we go beyond the analysis in this paper by developing a model of labor regulation and innovation with endogenous firm size distribution, that is matched with the data to obtain structural parameters, enabling us to perform aggregate counterfactuals.

Second, several structural papers look at the effects of labor regulations on employment and welfare, in particular Braguinsky, Branstetter and Regateiro (2011) on Portugal, Gourio and Roys (2014) and Garicano, Lelarge and Van Reenen (2016) on France. However, these papers do not allow for endogenous innovation, nor try to quantify the welfare changes arising form such dynamic considerations. More generally, there is a large literature focusing on how various kinds of distortions can affect aggregate productivity through the resulting misallocation of resources away from more productive firms and towards less productive firms. As Restuccia and Rogerson (2008) and Parente and Prescott (2000) have argued, these distortions imply that more efficient firms produce too little and employ too few workers. Hsieh and Klenow (2009) show that the resulting misallocation accounts for a significant fraction of the differences in aggregate productivity between the US, China and India and Bartelsman, Haltiwanger and Scarpetta (2013) confirm this finding using micro data from OECD countries.[7] Boedo and Mukoyama (2012) and Da-Rocha, Restuccia and Tavares (2019) have shown firing costs hinder job reallocation and reduce allocative efficiency and aggregate

[4]This is the same empirical variation used by Autor, Kerr and Kugler (2007) who actually found falls in TFP and employment from EPL. Bena, Ortiz-Molina and Simintzi (2020) finds a positive impact on process innovation using the same design.

[5]See also Cette, Lopez and Mairesse (2016) who document a negative effect of EPL on capital intensity, R&D expenditures and hiring of high skill workers. More generally, Porter and Van der Linde (1995) argue that some regulations, such as those to protect the environment, can have positive effects on innovation.

[6]Note that this is the opposite of what we find using our within-country identification. Labor regulation discourages low-value innovation, but has no impact on high-value innovation.

[7]In development economics many scholars have pointed to the "missing middle" , i.e. a preponderance of very small firms in poorer countries compared to richer countries (see Banerjee and Duflo, 2005, or Jones, 2011). Besley and Burgess (2000) suggest that heavy labor regulation in India is a reason why the formal manufacturing sector is much smaller in some Indian states compared to others.

productivity. The additional effect of barriers to reallocation when productivity is endogenous is also the focus of Gabler and Poschke (2013), Da-Rocha, Restuccia and Tavares (2019), and Bento and Restuccia (2017).[8] Mukoyama and Osotimehin (2019) is perhaps the most closely related paper to ours and finds a negative growth effect of the firing tax equivalent to a 5% labor tax (in the entrant-innovation model in the US) in a calibrated aggregate model with endogenous innovation. Unlike our approach, their paper does not have closed form solutions for the policy rules with taxes so has to rely on simulation methods. We contribute to this part of the literature by introducing an explicit source of distortion, namely the regulatory firm size threshold that goes beyond just firing costs, and by looking at how this regulation interacts with exogenous market size shocks using firm-level micro-econometric analysis.

Third, a body of work looks at the effects of EPL on the adoption of new technologies (e.g. Manera and Uccioli, 2020), especially information and communication technology. For example, Bartelsman, Gautier and De Wind (2016) argue that risky technologies require frequent adjustments of the workforce. By increasing the costs of such adjustments, EPL will deter technology adoption. Similarly Samaniego (2006) finds that EPL slows diffusion and Saint-Paul (2002) finds a smaller share of the economy in risky sectors when EPL are strong. Our approach is different as it focuses on technological innovation at the frontier rather than the adoption of existing technologies. Unlike emerging economies, advanced countries such as the US or France cannot rely solely on catch-up diffusion for long-run sustainable growth.

Fourth, our paper is related to public finance as we model regulation as an implicit tax, and a number of papers have examined how personal and business taxes affect innovation (see Akcigit and Stantcheva, 2020, for a recent survey). Like us, other tax papers use nonlinearities to identify behavioral parameters (e.g. Saez, 2010; Chetty et al., 2011; Kleven and Waseem, 2013; Kaplow, 2013 and Aghion et al., 2019*b*) and we contribute to this literature by bringing labor regulations, innovation and patenting into the picture.[9]

Fifth, there is an older literature looking at one form of labor regulation - union power - on innovation.[10] These papers found ambiguous theoretical and empirical effects . Finally, the heterogeneous effects of demand shocks on types of innovation is also a theme in the literature of the effects of the business cycle on innovation (Schumpeter, 1939; Shleifer, 1986; Barlevy, 2007). Recent work by

[8]Samaniego (2006) highlights the effects of firing costs in a model with productivity growth. He considers, however, only exogenous productivity growth and studies how the effects of firing costs differ across industries. Poschke (2009) is one of the few papers that study the effects of firing costs on aggregate productivity growth.

[9]This is important as Hopenhayn (2014) has argued that tax-driven reallocation distortions typically have only second order welfare effects unless there is rank reversal. Changing innovation is potentially a way of generating larger negative welfare effects that goes beyond static models.

[10]See Menezes-Filho, Ulph and Van Reenen (1998) for a survey and evidence. The common view is that the risk of ex post hold-up by unions reduces innovation incentives (Grout, 1984). But if employees need to make sunk investments there could be hold up by firms (this is the intuition of the Acharya, Baghai and Subramanian, 2013*a*,*b* papers).

Manso, Balsmeier and Fleming (2019) suggests that large positive demand shocks (booms) generate more R&D, but this tends to "exploitative" (incremental) rather than "exploratory" (radical) innovation. We find that the impact of regulation following a demand shock discourages incremental (but not radical) innovation.

The structure of the paper is as follows. Section II develops a simple model of how innovation can be affected by size-contingent regulation. Section III confronts the main qualitative predictions of the model to the data, using both a non-parametric cross-sectional analysis and a dynamic econometric analysis of the response to exogenous shocks. Section IV uses the theory and empirical moments (from both the static and dynamic analysis) to estimate the equilibrium effect of regulation on aggregate innovation and welfare. Section V presents a number of theoretical and empirical extensions and robustness tests, most importantly allowing for radical and incremental innovation. Section VI concludes. In Online Appendices, we present institutional details of the labor regulations (A), data details (B), further theoretical results (C) and additional empirical exercises (D).

II. Theory

In this section, we present our basic theory built around a discrete time version of the Schumpeterian growth model with firm dynamics by Klette and Kortum (2004) where we introduce size contingent regulations. This enables us to analytically characterize firms' innovation decisions depending on their size and the regulation. We next solve for the steady state firm size distribution incorporating both incumbent growth and entry/exit dynamics. Finally, we put both elements together to characterize how economy wide innovation changes with the stringency of the regulation. Throughout, we explore what the model implies for the steady state joint distributions of innovation and employment as well as how firms should respond to the exogenous demand shocks we will exploit in the empirical section.

A. A simplified Klette-Kortum model

We consider a simplified version of the two-period specification of Aghion et al. (2018*b*). We show that all results are theoretically and empirically robust to the longer lived owner model in the extension of subsection V.C. In the first period of her life, a firm owner decides how much to invest in R&D. In the second period, she chooses labor inputs, produces and realizes profits. At the end of the period, her offspring inherits the firm at its current size and a new cycle begins again.[11]

We assume that individuals have intertemporal log preferences:

$$U = \sum_{t>0} \beta^t \log(C_t), \tag{1}$$

[11] We do not consider bequest motives, but the extension to longer living owners which we present in Section V.C implicitly encompasses this incentive.

associated with a budget constraint:

$$w_t + (1 + r_t)a_t = a_{t+1} + C_t,$$

where w_t is the wage at time t, C_t is consumption, and a_t is an asset that yields an interest rate r_t. This immediately gives the Euler equation: $\beta(1+r_t) = 1+g_t$. We consider the economy on a balanced growth path where final output y and consumption grow at a constant rate which we denote by g, so that the Euler equation can be expressed as $\beta = (1 + g)/(1 + r)$, where β is the discount factor and r is the steady-state level of interest rate. There is a continuous measure L of production workers, and a mass 1 of intermediate firm owners every period. Each period the final good is produced competitively using a combination of intermediate goods according to the production function:

$$\ln y = \int_0^1 \ln(y_j)dj,$$

where y_j is the quantity produced of intermediate j. Intermediates are produced monopolistically by the firm who innovated last within that product line j, according to the linear technology $y_j = A_j l_j$ where A_j is the product-line-specific labor productivity and l_j is the labor employed for production. This implies that the marginal cost of production in j is simply w/A_j. A firm is defined as a collection of production units (or product lines/varieties) and expands in product space through successful innovation.

To innovate, an intermediate firm i combines its existing knowledge stock that it accumulated over time (n_i, the number of varieties it operates in) with its amount of R&D spending (R_i) according to the following Cobb-Douglas knowledge production function:

$$Z_i = \left(\frac{R_i}{\zeta y}\right)^{\frac{1}{\eta}} n_i^{1-\frac{1}{\eta}}, \tag{2}$$

where Z_i is the Poisson innovation flow rate, η is a concavity parameter and ζ is a scale parameter. This generates the R&D cost of innovation: $C(z_i, n_i) = \zeta n_i z_i^{\eta} y$, where $z_i \equiv Z_i/n_i$ is simply defined as the innovation intensity of the firm.

When a firm is successful in its current R&D investment, it innovates over a randomly drawn product line $j' \in [0, 1]$. Then, the productivity in line j' increases from $A_{j'}$ to $A_{j'}\gamma$ and the firm becomes the new monopoly producer in line j' and thereby increases the number of its production lines to $n_i + 1$. At the same time, each of its n_i current production lines is subject to the risk of being replaced by new entrants and other incumbents (a creative destruction probability that we denote x). Thus the number of production units of a firm of size n_i increases to $n_i + 1$ with probability $Z_i = n_i z_i$ and decreases to $n - 1$ with probability $n_i x$. A firm that loses all of its product lines exits the economy.

Because of the Cobb-Douglas aggregator, the final good producer spends the same amount y on each variety j. As a result, final good production function generates a unit elastic demand with respect to each variety: $y_j = y/p_j$. Combined with the fact that firms in a single product line compete *a la* Bertrand, this implies that a monopolist with marginal cost w/A_j will follow limit pricing by setting its price equal to the marginal cost of the previous innovator $p_j = \gamma w/A_j$.[12]

The resulting equilibrium quantity and profit in product line j are:

$$y_j = \frac{A_j y}{\gamma w} \text{ and } \Pi_j = \left(1 - \frac{1}{\gamma}\right) y,$$

and the demand for production workers in each line is given by $y/(\gamma w)$. Firm i's employment is then equal to its total manufacturing labor, aggregating over all n_i lines where i is active, N_i. Namely:

$$l_i = \int_{j \in N_i} \frac{y}{w\gamma} dj = \frac{y n_i}{w\gamma} = \frac{n_i}{\omega\gamma}, \tag{3}$$

where $\omega = w/y$ is the output-adjusted wage rate, which is invariant on a steady state growth path. Importantly for us, a firm's employment is strictly proportional to its number of lines n_i.

B. *Regulatory threshold and innovation*

We model the regulation by assuming that a tax on profit must be incurred by firms with a labor force that is greater than a given threshold $\bar{l}$ (50 in our application in France).[13] We suppose that $\bar{l}$ is sufficiently large that entrants do not incur this tax upon entry. There corresponds a cutoff number of varieties $\bar{n} = \bar{l}\omega\gamma$ to the employment threshold $\bar{l}$, such that if $n_i \geq \bar{n}$ profit is taxed at some additional positive marginal rate τ whereas the firm avoids this additional tax if $n_i < \bar{n}$.[14] Because firm owners live only for two periods, they can only expand the number of varieties of the firm by one extra unit during their lifetime. Hence, all the firms that start out with size $n_i < \bar{n} - 1$ or $n_i \geq \bar{n}$ act exactly as if the regulatory threshold did not exist. For firms that start with $n = \bar{n} - 1$, there is an additional cost to expanding by one extra variety.

[12]We implicitly assume a competitive fringe of firms with access to the previous technology in each sector; and that entering the market involves an ε entry cost. Then, as long as the new innovator sets a price which is less than the limit price equal to the marginal cost of fringe firms, no fringe firm will pay the entry cost. On the other hand, if the new innovator sets a price which is higher than the limit price, then she risks losing the market to a fringe firm.

[13]See Appendix A.3 for a discussion of alternative ways of modelling the regulation, such as including a fixed as well as a variable cost.

[14]Unlike in Aghion, Akcigit and Howitt (2014) where the innovation cost is modelled as a labor cost, here innovation uses the final good y as an input. With labor as R&D input, total employment is $l_i = \frac{n_i}{\omega\gamma} + \zeta n_i z_i^{\eta}$, and thus varies with innovation rather than being proportional to n_i. We consider this extension in subsection V.E where R&D is labor. Increased R&D will then affect the equilibrium wage.

The owner of a n-size firm therefore maximizes their expected net present value over its innovation intensity $z \geq 0$:[15]

$$\max_{z\geq 0}\left\{n\pi(n)y - \zeta n z^{\eta} y + \frac{1}{1+r}\mathbb{E}\left[n'\pi(n')y'\right]\right\},$$

where y' and n' denotes period 2's values for y and n and r is the interest rate. Dividing by y/n and using the fact that $\beta = (1+g)/(1+r)$, the above maximization problem can be re-expressed as:

$$\max_{z\geq 0}\Big\{\pi(n)(1+\beta) - \zeta z^{\eta} + \beta z[(n+1)\pi(n+1) - n\pi(n)] + \beta x[(n-1)\pi(n-1) - n\pi(n)]\Big\},$$

where $\pi(n) = \left(1-\frac{1}{\gamma}\right)$ if $n < \bar{n}$ and $\pi(n) = \left(1-\frac{1}{\gamma}\right)(1-\tau)$ if $n \geq \bar{n}$.

The intuition behind this equation is straightforward. The first term, $\pi(n)(1+\beta)$ represents the gross flow profits per line today and the second term is the cost of research, ζz^{η}. The third term, $\beta z[(n+1)\pi(n+1) - n\pi(n)]$, is the (discounted) incremental profit gain tomorrow multiplied by the probability the firm innovates and thereby operates one more product line. The final term, $\beta x[(n-1)\pi(n-1) - n\pi(n)]$ is the (discounted) incremental profits loss per line tomorrow if the firm gets replaced in one of its product lines by a rival firm.

Whenever positive, the optimal innovation intensity is therefore given by:

$$z(n) = \begin{cases} \left(\frac{\beta(\gamma-1)}{\gamma\zeta\eta}\right)^{\frac{1}{\eta-1}} & \text{if } n < \bar{n}-1 \\ \left(\frac{\beta(\gamma-1)(1-\tau\bar{n})}{\gamma\zeta\eta}\right)^{\frac{1}{\eta-1}} & \text{if } n = \bar{n}-1 \\ \left(\frac{\beta(\gamma-1)(1-\tau)}{\gamma\zeta\eta}\right)^{\frac{1}{\eta-1}} & \text{if } n \geq \bar{n} \end{cases} \tag{4}$$

Much of the core economics of the paper can be seen in equation (4). Innovation intensity, $z(n)$, is highest for small firms a long way below the threshold (first row on right hand side of (4)), second highest for large firms over the threshold (third row) and lowest for middle sized firms just to the left of the threshold (middle row).

[15]Since we have shown that innovation per line is the same for firms of a given size, we drop the firm i subscripts from here onwards for notational simplicity unless needed.

What we observe in the data is the firm's total innovation (measured using patents) which is $Z(n) = nz(n)$. Since employment is directly proportional to the number of product lines, this implies that the slope of the innovation-size relationship will depend upon whether the firm lies above or below the regulatory threshold. Typically, the upward sloping relationship between innovation and firm size should be steeper for small firms than for large firms and should fall and flatten discontinuously at the threshold. Furthermore, the ratio of the slopes of the innovation-size relationship for large versus small firms, relates directly to the underlying parameters of the model, and in particular upon the regulatory tax.[16] We will use this fact to empirically identify the magnitude of the regulatory tax, which we then use in our estimates of the aggregate impact of the regulation on innovation.

C. *Regulatory threshold and firm size distribution*

We now characterize the steady state distribution of firm size and look at how this distribution is affected by the regulatory tax. Let $\mu(n)$ be the share of firms with n lines. We first have a steady state condition saying that the number of exiting firms equals the number of entering firms in steady-state, namely: $\mu(1)x = z_e$, where z_e is the innovation intensity of entrants, which is the same as the probability of entry. Since x is the rate of creative destruction for any line, the number of exiting firms is therefore given by $\mu(1)x$.

For all $n > 1$, the steady state condition is that outflows from being a size n firm is equal to the inflows into becoming a size n firm. This can be expressed as:

$$n\mu(n)\left(z(n) + x\right) = \mu(n-1)z(n-1)(n-1) + \mu(n+1)x(n+1) \tag{5}$$

We know $z(n)$ for each n from equation (4) but we need to find the two remaining endogenous objects z_e and x. We close the model by considering the following two equations. First, the definition of μ gives $\sum_{n=1}^{\infty} \mu(n) = 1$. Second, the rate of creative destruction on each line is equal to the rate of creative destruction by an entrant plus the weighted sum of the flow probabilities $z(n)$ of being displaced by an incumbent of size n, namely:

$$x = z_e + \sum_{n=1}^{\infty} \mu(n)nz(n)$$

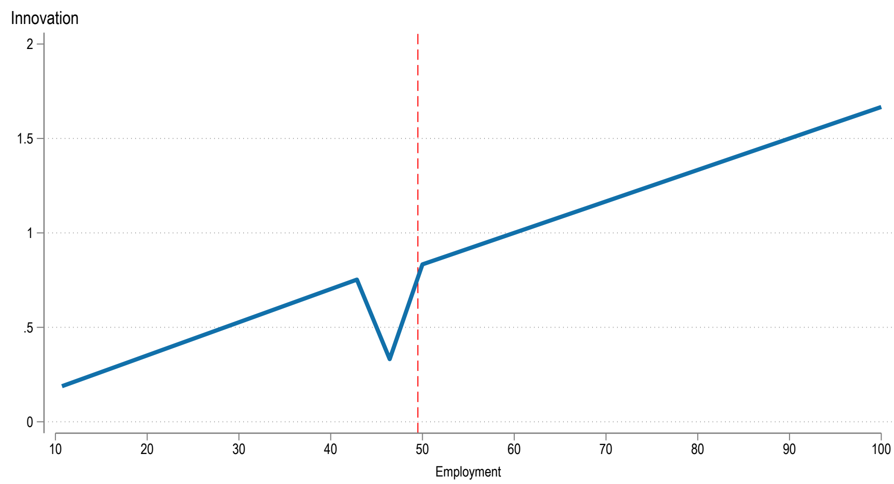
D. *Solving the model*

In Appendix C we detail how we solve the model numerically. The unknowns are $\mu(n)$ and $z(n)$ for all values of n as well as x and z_e, and the equations are

[16]The ratio of the innovation intensity of the first to third row in (4) is $(1-\tau)^{1/(1-\eta)}$. This can be empirically recovered from the relative slopes of the patents to size relationship before and after the regulatory threshold (see Section IV).

those derived above. To illustrate the effects, we first show firm-level innovation $Z(n) = z(n)n$ as a function of the firm's employment size $l = n/(\omega\gamma)$ in Figure 1. We see that firm-level innovation increases linearly with firm size until the firm nears the regulatory threshold, at which point there is a sharp innovation valley. After this, innovation again increases with firm size once the firm passes the threshold.

FIGURE 1. TOTAL INNOVATION BY FIRMS OF DIFFERENT EMPLOYMENT SIZES

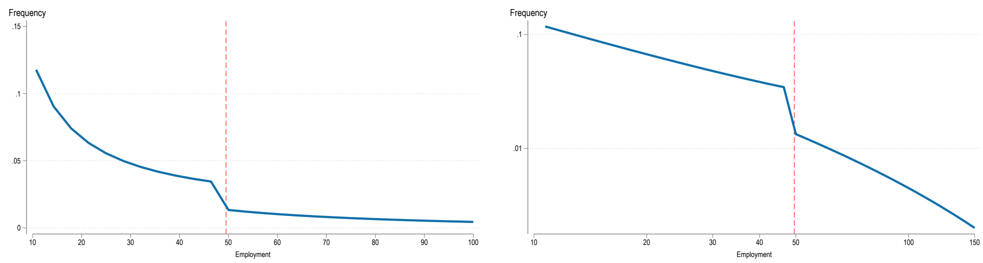


Note: This is the total amount of innovation ($Z(n)$) by firms of different sizes (employment, $l = n/(\gamma\omega)$) by aggregating innovation intensities $z(n)$ across all its product lines (n) according to our baseline theoretical model. The y-axis is the corresponding value of total innovation Z. We use our baseline calibration values of $\tau = 0.026$, $\gamma = 1.3$, $\eta = 1.5$, $\beta/\zeta = 1.70$ and $\omega = 0.22$ for illustrative purposes (see section IV for a discussion).

In Figure 2 we plot the equilibrium firm size distribution, i.e. the value of the density $\mu(n)$ for each level of firm employment. Panel (a) uses a linear scale, but because the distribution is nonlinear we plot it on a log-log scale in Panel (b) where it is broadly log-linear (the well-know power law as documented by Axtell, 2001 and many others). Note the departure from the power law around the regulatory threshold. The distribution bulges a bit as firms approach 50 and then discontinuously drops before falling again once firms pass the threshold. Unlike the innovation-size discontinuity, this "broken power law" in the French size distribution has been noted before in the literature (e.g. Ceci-Renaud and Chevalier, 2011), but the shape has proven difficult to fully rationalize in a model without endogenous innovation.[17]

[17]In particular, although a purely static model like Lucas (1978) with regulation can rationalize a discontinuity at 50 and a downwards shift of the line, there should be no firms of size 50 and no bulge at 48 (firms just fully shift to avoid the regulation and spike at 49). Garicano, Lelarge and Van Reenen

FIGURE 2. DISTRIBUTION OF FIRM SIZE ($\mu(n)$)



(a) Linear scale (b) Log scale

Note: These figures plot the density of firm employment, $\mu(n)$ according to our baseline theoretical model. Panel (a) uses a linear scale and Panel (b) uses a log-log scale. The calibration values are the same as Figure 1.

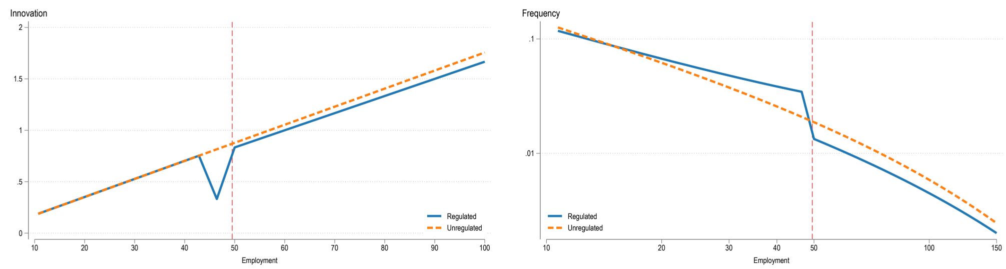
Although we took particular values of the parameters for illustrative purposes in Figures 1 and 2, these patterns are the same for any value of the regulatory tax (τ).[18] To see how τ qualitatively impacts the innovation-firm size relationship and the firm size distribution, we compare our results (solid blue) to an unregulated economy (i.e. $\tau = 0$, dashed red) in Figures 4(a) and 4(b). Four points are worth emphasizing. First, as expected, we observe no innovation valley when we remove the regulation Figures 4(a). Second, the level of innovation when $\tau > 0$ is lower than when $\tau = 0$ even for large firms to the right of the threshold. This stems from the fact that the tax reduces innovation intensity even for these firms. Third, the total innovation gap between the regulated and unregulated economy gets larger as firm size increases because bigger firms have more product lines and the innovation intensity of each line is lower than that of small firms. This can be seen from (4), which showed that the slope of the line after the threshold is flatter than that for small firms with $n < \bar{n} - 1$. Fourth, in terms of the size distribution in Figure 4(b), we see that moving from $\tau = 0$ to $\tau > 0$ increases the share of firms that are below 50 employees and decreases the fraction of large firms. The regulation also generates a larger mass of firms just below the threshold as these firms choose not to grow in order to avoid getting hit by the regulatory tax.

We now put together all the effects of regulation together to compute the overall impact of regulation on the economy-wide innovation, $\mathcal{Z}(\tau) = \sum_{i=1}^{\infty} \mu(i) z(i) i + z_e$. Figure 4 shows the fall in total innovation in the regulated economy compared to the counterfactual unregulated economy (where we normalize aggregate innovation at 1). The magnitude of the fall in innovation is clearly increasing in the

(2016) had to introduce *ad hoc* measurement error to rationalize the smoother bulge we see in the data around 45-50. This bulge (and the positive mass at 50) emerges more naturally with our dynamic endogenous innovation model.

[18]From equation (4), we know that we can take τ to lie anywhere between 0 and $1/\bar{n}$ in order to have an interior solution for $z(\bar{n})$.

Figure 3. Innovation and firm size distribution, with and without regulations



(a) Innovation (b) Size distribution (log scale)

Note: The blue solid line in this Figure reproduces Figure 1 in Panel (a) and Figure 3(b) in Panel (b). The orange dashed line is for an unregulated economy with all the same parameters in the regulated economy except $\tau = 0$.

intensity of the regulatory tax, τ. For example, there is a reduction in total innovation of 4% if $\tau = 0.02$ instead of zero. This fall in aggregate innovation comes from three sources. First, for a given firm size, the tax increase has a strong negative effect on innovation for firms just to the left of the threshold, and a smaller negative effect on innovation for all firms to the right of the threshold. Second, the tax increase reduces the mass of large firms, which are also the firms that do more innovation. Third, since lower incumbent innovation means less exit, this will mean there is less entry in steady state. When we use our data to quantify the model, we will decompose the fall of aggregate innovation into these different elements and show that the first element (incumbent innovation) dominates.

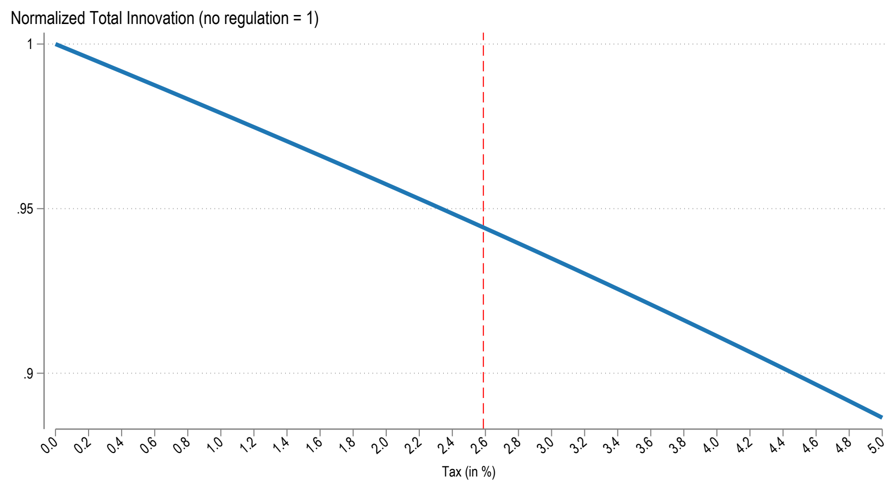
E. Effect of a demand shock

In the dynamic analysis below, we will examine the impact of market size shocks on innovation. In the theory, this can be seen as an exogenous idiosyncratic shock on the demand for one given product j.[19] Let us denote this shock by $\tilde{\varepsilon}_j$ which shifts the value of y_j for a given product j to $y_j(1+\tilde{\varepsilon}_j)$. The firm producing j anticipates that a shock will occur in next period but does not know in which product. As a result, the firm i expects a shock of magnitude $\varepsilon_i = \tilde{\varepsilon}_j/n_i$ in the demand for each of its products.

Given the expected demand shock, future expected profit is shifted by $(1+\varepsilon_i)$. Hence, holding innovation fixed, there will be a positive impact of ε_i on firm size in the short run, and this impact will be smaller for firms just to the left of the threshold as these firms will not want to cross the threshold and bear the extra regulatory cost.

[19] A common shock to all firms can be modeled as an increase in y. This will not have a differential effect on innovation in firms of different size, as all variables in our model are expressed in units of final output.

FIGURE 4. AGGREGATE ECONOMY-WIDE INNOVATION AS A FUNCTION OF THE INTENSITY OF REGULATION



Note: We simulate the amount of aggregate innovation in different economies relative to an unregulated benchmark economy as the intensity of regulation changes as indicated by the magnitude of the implicit tax (τ). For example, if $\tau = 2\%$, aggregate innovation is about 0.96 relative to the benchmark, i.e. 4% lower. Parameter values are the same in regulated and regulated economies (as in notes to Figure 1) except we vary the value of τ. The vertical line corresponds to our baseline estimate of τ (see Table 3).

What is the effect of the impact of the shock on firm-level innovation? Equation (4) is modified by having the shock factor $(1+\varepsilon_i)^{\frac{1}{\eta-1}}$ pre-multiplying each term of the equation. Formally, the value of Z becomes (for $n \neq \bar{n}-1$):

$$Z(n,\varepsilon) = \left(\frac{\beta\pi(n)}{\zeta\eta}\right)^{\frac{1}{\eta-1}} \omega\gamma l(n)(1+\varepsilon)^{\frac{1}{\eta-1}}$$

where $l(n) = n/(\omega\gamma)$ is the level of employment without a shock. Hence, a shock ε implies a change in Z such that:

$$\Delta Z(n,\varepsilon) \equiv Z(n,\varepsilon) - Z(n,0) = \left(\frac{\beta\pi(n)}{\zeta\eta}\right)^{\frac{1}{\eta-1}} \omega\gamma l(n)\left[(1+\varepsilon)^{\frac{1}{\eta-1}} - 1\right] \tag{6}$$

The impact of the shock on innovation intensity will be largest for small firms far below the regulatory threshold. The second biggest effect will be on innovation in large firms well to the right of the threshold. And the smallest effect of the demand shock will be on firms just below the threshold. The overall increase in a firm's innovation (number of lines multiplied by the innovation intensity per line) in response to the shock will be greater for large firms as they have more product lines. However, even controlling for firm size (as we will do in the empirical work), and so concentrating on the marginal effect of the shock on innovation intensity, the model predicts that the effect of a market size shock on innovation should be significantly lower for firms just to the left of the threshold because:

$$\frac{\partial^2 \Delta Z}{\partial \varepsilon \partial l} = \left(\frac{\beta\pi(n)}{\zeta\eta}\right)^{\frac{1}{\eta-1}} \frac{\omega\gamma}{\eta-1}(1+\varepsilon)^{\frac{2-\eta}{\eta-1}},$$

which continues to depend upon $\pi(n)$, the profit per line of a firm of size n.

Finally, the shock will affect the firm size distribution. If the shock is transitory, a shocked firm will grow larger for a short period of time before the economy will return to the initial steady state distribution. A permanent idiosyncratic shock will translate into a permanent change to the overall steady state size distribution. The dynamic empirical design is not well suited to analyzing the impact on the steady state firm size distribution as the Bartik-style shock is defined only for incumbents. Hence, we focus on entry effects only in the equilibrium calibration.

III. Empirics

We have combined multiple administrative datasets on firm employment size, innovation and trade. This will enable us to examine the predictions from the theory both statically (e.g. cross sectional distribution of firm innovation by firm size) and dynamically - i.e. how firm innovation responds heterogeneously across the size distribution to the same market size shock due to the regulation.

A. Data

Our main data comes from the French tax authorities, which consistently collect information on the balance sheets of all French firms on a yearly basis from 1994 to 2007 ("FICUS", Insee & DGFiP, 1994). We restrict attention to non-government businesses and take patenting information from Lequien et al. (2017). This matches the PATSTAT (Spring 2016 version, EPO, 2016) database to FICUS using an algorithm, which matches the name of the assignee - the holder of the IP rights - on the patent front page to the firm whose name and address is closest to that of the patent holder. The accuracy of the algorithm is worse for firms that are below 10 employees so we focus on firms with more than 10 employees. Since we are interested in the effects of a regulation that affects firms as they pass the 50 employees threshold, we further restrict attention to firms with between 10 and 100 workers in 1994 (or the first year those firms appear in the data).[20] Other data sources are used to calculate the demand shocks, in particular BACI (Gaulier and Zignago, 2010) and the custom data (French customs and indirect taxation authorities (DGDDI), 2023). More details about the data source are

[20]We show robustness of the results to changing this bandwidth (see in particular Table D2 in Appendix D).

given in Appendix B.[21]

Our main sample consists of 182,347 distinct firms and 1.66 million observations. We report some basic descriptive statistics in Table 1 for all firms in all parts of the market economy in Panel A (25% are in manufacturing) and for the sub-sample of firms who filed at least patent between 1994-2007 in Panel B. We can see that on average, firms file 0.009 patents per year and, conditional on being an innovator, 0.28 per year. As is well known, the distribution of innovation is highly skewed, with a small number of firms owning a large share of the patents in our sample. However, since we do not include the largest French firms in our data, the skewness is less pronounced.

Table 1—Descriptive statistics

Panel A: All firms	Mean	p25	p50	p75	p90	p99
Employment	29	12	20	35	56	151
Sales	5,434	958	2,032	4,756	10,632	45,224
Patents	0.0090	0	0	0	0	0
Innovative	0.031	0	0	0	0	1
Manufacturing	0.25	0	0	1	1	1

Panel B: Subset of innovative firms	Mean	p25	p50	p75	p90	p99
Employment	52	21	37	62	98	307
Sales	11,795	2,500	5,208	10,492	21,326	115,145
Patents	0.283	0	0	0	1	4
Manufacturing	0.68	0	1	1	1	1

Note: These are descriptive statistics on our data. Panel A is all firms and Panel B conditions on firms who filed for at least one patent between 1994 and 2007 ("Innovative" firms). We restrict to firms who have between 10 to 100 employees in 1994 (or the first year they enter the sample). There are 182,347 firms and 1,658,760 observations in Panel A and 4084 firms and 51,192 observations in Panel B.

B. *Nonparametric evidence: Static Analysis*

Figure 5 shows, for each employment size bin, the fraction of firms within that bin with at least one patent (see also Panel A of Table 1). We see an almost linear relationship between firm size and the fraction of innovative firms. That larger

[21] Access to the data and guidelines to replicate our results is given in the replication package. The matching between patents and firms from Lequien et al. (2017) is available upon request.

firms are more likely to patent is in line with the analysis in Akcigit and Kerr (2018). The prediction of a linear relationship between firm size and innovation is consistent with our equation (4).

For firms just below the 50 employee threshold, the share of innovative firms suddenly decreases in an innovation valley. This is what the model predicts. It is also noteworthy that the slope of the innovation-size relationship is flatter for larger firms to the right of the threshold than for smaller firms below the threshold. This again is consistent with our theoretical predictions. Note that in the theory, the ratio between the slopes of the innovation-size relationship between a large and a small firm, varies with the tax (τ) and with the concavity of the R&D cost function (η). We will exploit this variation to help recover the tax parameter later in this section.[22]

The innovation outcome measure is taken over the whole sample period from 1994 to 2007, but the same patterns emerge if we consider alternative definitions of an innovative firm (see Appendix Figure D2). The predictions over the size distribution also broadly match up to the data, but since these are relatively well known we relegate discussion to Appendix D.

C. Dynamic analysis

Estimation equation

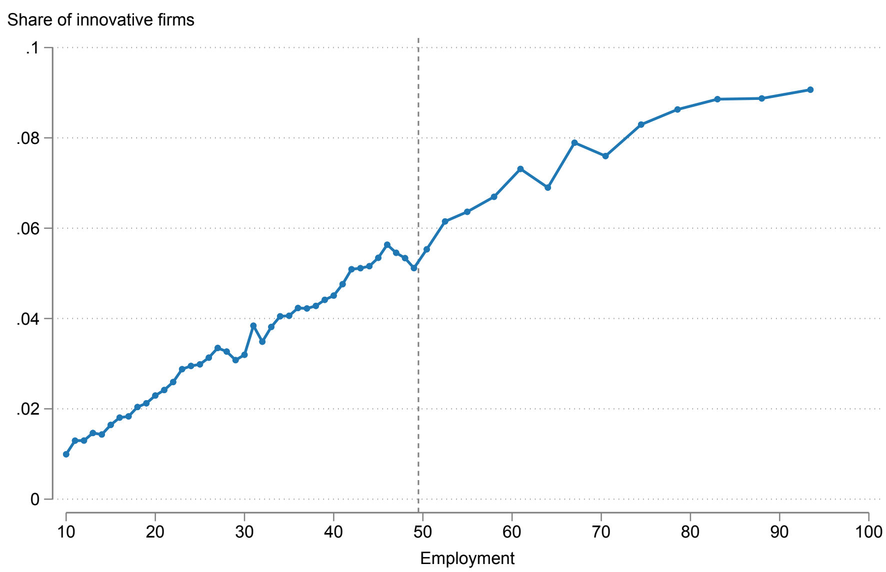
Recall that the theoretical response of innovation to demand shock in the model is given by (6). As discussed in Section II.E, the first derivative of ΔZ with respect to the shock will depends upon the value of l, and this value will depend on whether the firm is close to the threshold, far below it, or far above it. The second derivative of ΔZ with respect to employment and the shock will be significantly lower for firms located in the innovation valley.

We take this into account in our empirical exploration of the effect of a demand shock and turn to a parametric investigation of how firms respond to market size shocks by considering the following regression:

$$\begin{aligned} \tilde{\Delta} Y_{i,t} = \; & b_1 l^{\star}_{i,t-2} + b_2[\Delta S_{i,t-2} \times \mathcal{P}(log(l_{i,t-2}))] \\ & + b_3[\Delta S_{i,t-2} \times l^{\star}_{i,t-2}] + \phi \mathcal{P}(log(l_{i,t-2})) \\ & + \psi_{s(i,t),t} + \epsilon^{err}_{i,t} \end{aligned} \tag{7}$$

[22] A concern with this approach is that the flattening of the innovation-size gradient could occur for non-regulatory reasons. For example, Akcigit and Kerr (2018) argue that larger firms invest in more 'internal' R&D to protect their market share that generates less knowledge than the 'external' R&D of smaller firms. We tackle this issue in two ways. First, we will look at the aggregate innovation losses using the dynamic moments derived in the next section that analyzes the responsiveness to shocks rather than just the cross sectional moment looking at levels in Figure 5. Second, we confirmed that the flattening of the gradient in Figure 5 does not seem to occur in micro-datasets from the UK and US (countries which do not have the large increase in labor regulations for firms with 50 or more employees).

Figure 5. Share of innovative firms at different levels of employment



Note: Share of innovative firms (i.e. with at least one priority patent) plotted against their employment. All observations are pooled together. Employment bins have been aggregated so as to include at least 10,000 firms. The sample is based on all firms with initial employment between 10 and 100 (182,347 firms and 1,658,760 observations, see Panel A of Table 1).

where $Y_{i,t}$ is a measure of innovation (based on patents) that is related to Z in the theory and $l_{i,t}$ a measure of employment. $\Delta S_{i,t-2}$ is an exogenous demand shock to market size that should trigger an increase of innovation in a wide class of models (and in our own, is related to the demand shock ε) and $\psi_{s(i,t),t}$ is a set of industry-year dummies where $s(i,t)$ denotes the main sector of activity of firm i at time t.

Our main focus is to see whether there is a discouraging effect of the regulation on innovation. For this reason, we include $l^{\star}_{i,t}$ in the model, a binary variable that takes value 1 if firm i is close to, but below, the regulatory threshold at time t. Our baseline measure of $l^{\star}_{i,t}$ is a dummy for a firm having employment between 45 and 49 employees. In this specification, in order to capture the heterogeneous response across the different values of employment predicted by the model, we use a flexible functional form and include $\mathcal{P}(\log(l_{i,t-2}))$ a polynomial in $\log(l_{i,t-2})$. Finally, $\epsilon^{err}_{i,t}$ is an error term. We use a two year lag of the shock since there is likely to be some delay between the demand shock, the increase in research effort and the filing of a patent application.

Finally, for the dependent variable, we need a data equivalent to ΔZ. We proxy Z as the log of the number of patents, and measure its growth by:[23]

$$\tilde{\Delta} Y_{i,t} = \begin{cases} \frac{Y_t - Y_{t-1}}{Y_t + Y_{t-1}} & \text{if } Y_t + Y_{t-1} > 0 \\ 0 & \text{otherwise} \end{cases}$$

The key coefficient capturing the discouragement effect of the threshold in equation (7) is b_3, which we expect to be negative. Larger firms will likely respond more to a given shock, but this relationship should break down for the firms just to the left of the threshold as firms are reluctant to cross the threshold in response to an expansion in market size.

Market Size Shocks

To construct the innovation shifters $\Delta S_{i,t-2}$, we rely on international trade data to build export demand shocks following Hummels et al. (2014) and Mayer, Melitz and Ottaviano (2016). In short, we look at how foreign demand in a given product by destination cell changes over time by measuring the change in imports from all countries (except France) into that product-country cell. We then build a product-destination portfolio for each French firm i, and weight the foreign demands for each product by the relative importance of that product for firm i.

[23] This is essentially the same as in Davis and Haltiwanger (1992) for employment dynamics except that we set the variable equal to zero when a firm does not patent for two periods. The results are robust to considering other types of growth rates such as using the Inverse Hyperbolic Sine (see columns 7-9 of Table D2 in Appendix D).

More specifically, firm i's export demand shock at date t is defined as:

$$\Delta S_{i,t} = \sigma_{i,t_0} \sum_{s,c\in\Omega(i,t_0)} \omega_{i,s,c,t_0} \tilde{\Delta} I_{s,c,t}, \tag{8}$$

where $\Omega(i, t_0)$ is the set of products and destinations associated with positive export quantities by firm i in the first year t_0 in which we observe that firm in the customs data[24] and ω_{i,s,c,t_0} is the relative importance of product s and country destination c for firm i at t_0, defined as firm i's exports of product s to country c divided by total exports of firm i in that year. $I_{s,c,t}$ is country c's demand for product s, defined as the sum of its imports of product s from all countries except France and σ_{i,t_0} is the initial export intensity (export divided by sales) of firm i. The basic idea behind the shock design is simply that a firm that was exporting, for example, many cars to China in 2000, would have benefited disproportionately from the boom in Chinese consumption of cars at the start of the twenty-first century.[25]

We fix the weights at the firm level taking initial period t_0 as the reference. This is done in order to exclude any variation in the portfolio of products and countries that could be endogenous. Our shock is therefore similar to a "Bartik"-type shift-share instrument. There is an important recent literature (e.g. Goldsmith-Pinkham, Sorkin and Swift, 2020 and Adao, Kolesár and Morales, 2019) which discusses inference and estimations with these designs. In particular, the sum of exposure weights across (s, j)'s is not 1 (because of σ_{i,t_0}, except in the rare case of firms that do not sale domestically) and varies across-firms. We follow Borusyak, Hull and Jaravel (2018) who argue that in such an "incomplete shift-share" case with panel data, it is important to control for this sum and allow the coefficient to change with time.[26]

[24]French customs data are available from 1994. So we use 1994 as the initial year, except for firms who enter after 1994 for which we use the initial year they enter the sample.

[25]We clean $I_{s,c,t}$ to remove extreme values due to trade disruption because of wars, for example. To do so, we follow Aghion et al. (2018*b*) and look at the within product-country standard deviation of $\tilde{\Delta} I_{s,c,t}$, winsorizing values of $\tilde{\Delta} I_{s,c,t}$ that are above the 90% percentile. This mostly concerns pairs of country-product where French firms do not export and this impacts less than 0.15% of total observations. We then trim the shock $\Delta S_{i,t}$ at the 0.5 level. This procedure has has no material impact on our results (for example, see Table D2, column 10 in the Appendix).

[26]We have conducted many more extensive diagnostic tests showing the validity of this source of exogenous variation to market size. Borusyak, Hull and Jaravel (2018) underline two assumptions underlying the validity of a shift-share instrument: quasi-randomness of shock assignment and a high number of uncorrelated shocks. The first assumption is likely to hold in our setting due to the inclusion of narrow industry by year dummies in our regressions. The assumption is essentially that within industry, the expected value of $\tilde{\Delta} I$ is the same for all firms conditional on the country-product-level unobservables. The second assumption is warranted by the fact that we consider a very large number of shocks across many countries and products. In one robustness test, we follow the recommendations of Borusyak, Hull and Jaravel (2018) and check that our main results are robust to using alternative shocks in which $\tilde{\Delta} I$ has been residualized on different combinations of year, country, product fixed effects. Moreover, note that our panel data structure allows us to include a firm fixed effect as an additional robustness check which further controls for potential correlations between permanent firm characteristics and future realizations of the shocks. See Aghion et al. (2018*a*) for more diagnostics.

Testing the main prediction

To estimate equation (7), we need to make some further restrictions in our use of the dataset. First, note that the market size shock ΔS is only defined for exporting firms, that is, firms that appear at least once in the customs data from 1994 to 2007. Second, in order to increase the accuracy of our shock measure, we restrict attention to the manufacturing sector. Not only is a large fraction of patenting activity located in manufacturing, but these firms are also more likely to take part in the production of the goods they export (see Mayer, Melitz and Ottaviano, 2016). Our main regression sample is therefore composed of 20,640 firms and 142,560 observations.

Table 2—Main regression results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$Shock_{t-2} \times L^{\star}_{t-2}$				-5.103**	-6.159***	-5.661**	-6.682**	-3.716**	-6.158**
				(2.185)	(2.178)	(2.221)	(2.710)	(1.769)	(2.304)
$L^{\star}_{t-2}$				0.052	0.078	0.078	0.003	0.086	0.078
				(0.109)	(0.132)	(0.111)	(0.193)	(0.056)	(0.113)
$Shock_{t-2}$	1.082**	-4.743**	7.219	1.401**	-5.268**	5.931	-5.635**	-3.309**	-5.291**
	(0.488)	(2.018)	(6.380)	(0.509)	(2.512)	(6.275)	(2.368)	(1.249)	(2.040)
$log(L)_{t-2}$		-0.048	-0.015		-0.053	-0.026	-0.114	-0.037	-0.057
		(0.038)	(0.162)		(0.035)	(0.162)	(0.188)	(0.024)	(0.034)
$Shock_{t-2} \times log(L)_{t-2}$		1.738**	-5.984		2.009**	-5.214	2.146**	1.213***	2.014***
		(0.637)	(4.608)		(0.819)	(4.532)	(0.798)	(0.441)	(0.660)
$log(L)^2_{t-2}$			-0.006			-0.005			
			(0.029)			(0.029)			
$Shock_{t-2} \times log(L)^2_{t-2}$			1.166			1.088			
			(0.760)			(0.749)			
$\Delta log(L)_{t-2}$									0.046
									(0.224)
Fixed Effects									
Sector× Year	✓	✓	✓	✓	✓	✓	✓	✓	✓
Firm							✓		
Number Obs.	142,560	142,560	142,560	142,560	142,560	142,560	141,071	330,423	142,479

Note: This contains OLS estimates of equation (7) on the manufacturing firms in Panel A of Table 1 who have exported at some point 1994-2007. Dependent variable is the Davis and Haltiwanger (1992) growth rate in the number of priority patent applications between $t-1$ and t. Column 1 only considers the direct effect of the shock, taken at $t-2$, column 2 uses a linear interaction with $log(L)$ taken at $t-2$ and column 3 considers a quadratic interaction. Columns 4, 5 and 6 do the same as columns 1, 2 and 3 respectively but also includes an interaction with $L^{\star}$, a dummy variable for having an employment size between 45 and 49 employees at $t-2$. Column 7 replicates column 5 but adds firm fixed effects. Column 8 includes non-manufacturing firms and column 9 also controls for the growth in log(employment) at $t-2$. All models include a 2-digit NACE sector interacted with a year fixed effect and a time fixed effect interacted with the initial level of export intensity. Estimation period: 1998-2007 . Standard errors are clustered at the 2-digit NACE sector level. ***, ** and * indicate p-value below 0.01, 0.05 and 0.1 respectively.

Table 2 presents the results from estimating equation (7), i.e. from regressing the growth rate of firm patents on the lagged market size shock. Column (1) shows that firms facing a positive exogenous export shock are significantly more likely to increase their innovative activity. The coefficient implies that a 10%

increase in market size increases patents by about 1.1%. Column (2) includes a control for the lagged level of log(employment) and also its interaction with the shock. The interaction coefficient is positive and significant, indicating that there is a general tendency for larger firms to respond more to the shock than smaller firms. Although it is not of direct interest, this is what we should expect given our discussion in II.E. Column (3) generalizes this specification by adding in a quadratic term in lagged employment and its interaction with the shock.

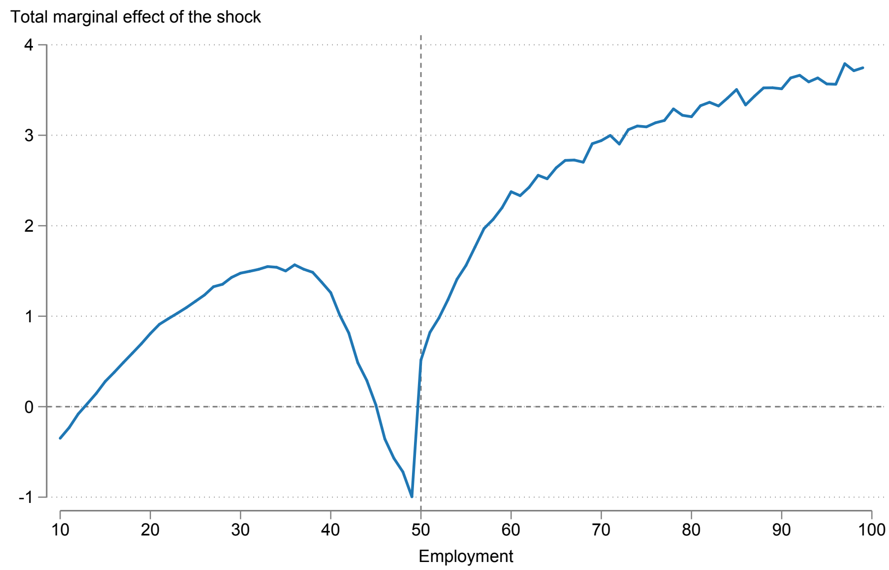
Column (4) of Table 2 returns to the simpler specification of column (1) and includes a dummy for whether the firm's employment is just below the regulatory threshold in the 45-49 employees range (defined as $l^{\star}$) at $t-2$, and the interaction of this dummy with the shock. Our key coefficient is on this interaction term, and it is clearly negative and significant as our model implies. This is one of our key results: innovation in firms just below the regulatory threshold is significantly less likely to respond to positive demand opportunities than in firms further away from the threshold. Our interpretation is that when a firm gets close to the employment threshold, it faces a large "growth tax" due to the regulatory cost of becoming larger than 50 employees. Consequently, such a firm will be more reluctant to invest in innovation in response to this new demand opportunity. The firm might even simply cut its innovative activities altogether to avoid the risk of crossing the threshold. We depict the relationship between innovation and the shock in Figure 6. The figure plots the implied marginal effect of the market size shock on innovation (at $t-2$) for different firm sizes (at t) using the coefficients in column (5) of Table 2. We see that innovation in larger firms tends to respond more positively to the market size shock than in smaller firms, but at the regulatory threshold there is a sharp fall in the marginal effect of the demand shock, consistent with our model (e.g. see subsection II.E).

It might be the case that the negative interaction between the threshold and the shock could be due to some omitted non-linearities. Hence in column (5) we also include lagged employment and its interaction with the shock (as in column (2)). These do have explanatory power, but our key interaction coefficient remains significant and negative and we treat this as our preferred specification. Column (6) adds a quadratic employment term and its interaction following column (3). Our key interaction remains significant and these additional non-linearities are insignificant.

Robustness of the dynamic empirical model

We have subjected our results to a large number of robustness tests, many of which are detailed in Appendix D. Column (7) of Table 2 shows the results from a tough robustness test where we include a full set of firm dummies. Given that the regression equation is already specified in first differences, this amounts to allowing firm-specific time trends. The key interaction between the market size shock and the threshold dummy remains significant. The data sample underlying Table 2 is limited to manufacturing firms, so column (8) also adds in non-manufacturing

Figure 6. Marginal effect of a market size shock on innovation



Note: marginal effect of a shock at $t-2$ at different level of employment at t, based on the model in column 5 of Table 2.

firms. The relationship remains negative, though with a smaller coefficient. This is likely to be due to the fact that patents are a much more noisy measure of innovation in non-manufacturing firms. We also experimented with including manufacturing firms who do not export by using the industry-level equivalent of our market size shock in equation (8). The coefficient on the key interaction remained negative and significant in column (6) of Table D2.

Does the number of patents grow more slowly for firms to the left of the threshold who experience a demand shock simply because their employment grows by less? Column (9) of Table 2 provides a crude test of this hypothesis by including the growth of employment on the right hand side of the regression. This variable is endogenous, of course, yet it is interesting to observe from a purely descriptive viewpoint that the interaction between the market size shock and the threshold remains significant. This suggests that it is patenting per worker, which is reacting negatively to the interaction between the shock and the threshold: our effect on patenting is not simply reflecting differential changes in firm size.

IV. The aggregate effects of regulation on Innovation

So far, we have established that many of the *qualitative* predictions of our simple model are consistent with the data both from a cross-sectional analysis and a more challenging dynamic analysis of the response to shocks. In this section, we use the data, the structure of our theoretical model and some external calibration values to estimate the general equilibrium effects of the regulation on aggregate innovation and welfare. This clearly requires stronger assumptions as we are extrapolating well away from the threshold.[27] Our baseline approach uses static moments from the non-parametric analysis covering the whole private sector. But in an extension we consider using the dynamic estimates from the exporting manufacturing sub-sample to calibrate the implicit cost of the regulatory tax.

A. *Quantitative Strategy*

We sketch some of the important elements here. The threshold number of product lines, $\bar{n}$, can be calculated from the known regulatory employment threshold of 50, i.e. $\bar{n} = 50\omega\gamma$ (see equation (3)), so we have six unknown parameters: $(\eta, \omega, \gamma, \beta, \zeta, \tau)$. Since we only need the ratio β/ζ to calculate the aggregate innovation loss, we only need to quantify five parameters $(\eta, \omega, \gamma, \beta/\zeta, \tau)$. We use the existing literature to obtain two of them (η and γ) and the remaining three are chosen to match moments from the data as detailed in Table 3.

Table 3—Calibration values and moments

Parameter	Value	Name	Source
From the literature			
η	1.5	Concavity of Innovation cost function	Dechezlepretre et al. (2016)
γ	1.3	Productivity step size	Aghion et al. (2019*a*)
Using our data			
τ	0.0259	Regulatory tax	Innovation-Firm size relationship $(\hat{\beta}_1, \hat{\beta}_2)$
β/ζ	1.70	Discount factor/scale parameter	Long-term growth of GDP
ω	0.22	Output adjusted wage	Gap in the firm size distribution

Note: Long-term growth of GDP is taken from the national accounts and corresponds to the average growth between 1990 and 2019 (1.63%). The gap in the first size distribution is estimated by dividing the number of firms at 40-45 employees by the number of firms of size 50-55 in 2000 and is equal to 3.05. See Figure D1.

Concavity of the R&D cost function η

In order to calibrate the concavity of the R&D cost function, η, we draw upon existing work that has estimated the innovation production function (the relationship between patents and R&D). Acemoglu et al. (2018) use a value of $\eta = 2$ based on Blundell, Griffith and Windmeijer (2002). However, these estimates typically come from very large US firms (publicly listed companies from Compustat), so

[27]A conservative approach would be to say we calculate aggregate losses for the sub-sector of the economy with firms under 100 employees (about 50% of all jobs are in such firms in France). However, Appendix D shows that our findings are robust to extending the sample to firms of up to 250, 500 or even 1,000 employees. Given this, the labelling of our estimates as “macro-economic” appears reasonable.

may exaggerate η, which is likely to be lower for the small and medium sized enterprises that are the vast bulk of our sample.[28] The estimates of Dechezlepretre et al. (2016) look at firms of similar sizes to the ones we use here, suggest a value of $\eta = 1.5$, using their Regression Discontinuity Design, which should produce cleaner causal estimates of the impact of R&D on innovation. This value is also consistent with some of the estimates in Crépon and Duguet (1997) on French firm panel data.

Regulatory tax τ

To quantify the regulatory tax (τ), we estimate empirically the changing slope of the relationship between innovation and firm size from equation (4). Our theory implies that the ratio of the innovation-size slope for small firms (before the innovation valley) to large firms (to the right of the regulatory threshold) should be equal to $(1-\tau)^{\frac{1}{\eta-1}}$. In other words, for any given value of η, a larger tax will mean a greater flattening of the positive relationship between innovation and firm size. Figure 5 shows this flattening very clearly and we recover this through a simple regression of patents on lagged size for firms under 45 employees and firms firms over 50 employees (to abstract from the innovation valley), allowing the coefficient on size to be different for these two size groups. Empirically, we average the number of patent applications filed by a firm over a five-year window for each possible value of employment l between 10 and 100. Our baseline estimation uses the same mapping between Z and patents as in Section III, i.e. we measure Z using the logarithm of the number of patents. We then jointly estimate two slopes for $L \in [10; 45)$ and $L \in [50; 100]$. We respectively denote $\hat{\beta}_1$ and $\hat{\beta}_2$ the OLS estimate of these two slopes. We find $\hat{\beta}_1/1000 = 1.801$ with a standard error ($\hat{\sigma_1}/1000$) of 0.105 and $\hat{\beta}_2/1000 = 1.709$ with a standard error ($\hat{\sigma_2}/1000$) of 0.0381. Hence, according to our model we have:

$$\frac{\hat{\beta}_2}{\hat{\beta}_1} = (1-\tau)^{\frac{1}{\eta-1}} = 0.949 \tag{9}$$

Given the calibrated value of $\eta = 1.5$ this yields an estimate of $\tau = 0.0259$, a regulatory tax of 2.59 percent. There are several ways to estimate this slope and we discuss the sensitivity to the choice of alternative empirical models extensively in Appendix D.4. Alternative models generate implicit taxes in the range of 1% to 5%, so we are effectively choosing a calibration value just below the midpoint of this range.

[28]Labelling the estimated elasticity between patents and R&D as θ, $\eta = 1/\theta$. Since θ is likely larger for small firms (e.g. due to financial constraints) or in countries with less developed risk capital markets (e.g. France vs. the US) this implies a smaller η.

Step size γ

The productivity step size γ following innovation is set to 1.3 using based on estimates in Aghion et al. (2019*a*). This is derived from various estimates of the average markup, which in our model is the reward from innovation.

Productivity adjusted wage rate ω

A larger ω means a higher cost of labor and therefore a smaller mass of large firms. Therefore to set the value of ω, we use the empirical firm size distribution. In particular, we match the fall in the density of employment of smaller vs. larger firms to the left and right of the innovation valley. In our data there are about three times as many firms between 40 and 45 employees than between 50 and 55 and the value of ω that reproduces this gap is 0.22.

Scale parameter and discount factor β/ζ

We calibrate β/ζ in order to match the measured value of g in the data that we take to be equal to the average growth of GDP in France over the period 1990-2019 (1.63%). In our model, growth g is defined as follow:

$$g = \exp\left(\left[z_e + \sum_{i=1}^{N} \mu(i)z(i)i\right]\log(\gamma)\right) - 1.$$

This yields a value of β/ζ of 1.70.

B. *Results*

Measuring and Decomposing Innovation Losses: Baseline Estimates

Plugging in these quantitative estimates of the key parameters implies a loss of aggregate innovation of about 5.7% percent compared to the no regulation benchmark (see the first row of Table 3). The implied regulatory tax of $\tau = 0.0259$, is the key parameter as can be seen from Figure 4. Since this maps back into growth rates, it means that the steady state growth rate in France would rise from its current average annual rate of 1.62% to 1.72%, a nontrivial change. As discussed in the modeling section, the aggregate innovation loss is driven by three major elements:

1) The decline in the incumbent innovation rate ($z(n)$) for a given firm size. For any given size distribution of firms, the regulation reduces innovation rates for firms above the threshold and just to the left of the threshold.

2) The change in the size distribution μ. Since the regulation pushes the size distribution to the left and smaller firms do less innovation, this reduces aggregate innovation.

3) The decline in the innovation rate by entrants z_e.

Recall that we have denoted $\mathcal{Z}(\tau) = \sum_{i=1}^{\infty} \mu(i)z(i)i + z_e$ total innovation in the economy when the regulation tax is set to τ and the value of other variables are taken from Table 3. Analogously to a shift-share decomposition analysis we have:

$$\begin{aligned} \mathcal{Z}(\tau) - \mathcal{Z}(0) &= \sum_{n>0} \left(Z(n,\tau) - Z(n,0)\right) \mu(n,0) \\ &+ \sum_{n>0} \left(\mu(n,\tau) - \mu(n,0)\right) Z(n,0) \\ &+ \sum_{n>0} \left(\mu(n,\tau) - \mu(n,0)\right) \left(Z(n,\tau) - Z(n,0)\right) \\ &+ z_e(\tau) - z_e(0), \end{aligned} \tag{10}$$

where $\mu(n, \tau)$ and $Z(n, \tau)$ are the share of firm of size n where the economy has a regulation tax of τ and their total innovation respectively. The first term in the right hand side of equation (10) is the innovation intensity (evaluated at the size distribution in the unregulated economy) and the second term is the effect on size (evaluated at a firm's innovation intensity rate in the unregulated economy). The third term is the interaction effect between the first two terms and the final term is the effect on entrants (since an entrant must innovate by definition to displace an incumbent).

Dividing equation (10) by $\mathcal{Z}(0)$, we can have an approximation of where the 5.7%% loss of aggregate innovation comes from. We find that most (80%) of the effect comes from the change in the innovation intensity (the first term in the right hand side of the previous equation). The covariance and entry terms (third and last terms) account for roughly 10% each (9.6% and 10.3% respectively), while the change in the size distribution has almost no effect. The virtual absence of any effect of the size distribution is due to the relatively small value of the tax.

Robustness of the Baseline Aggregate Calculations

We now explore how the 5.7% loss in innovation is affected when we consider variations in the parameters from Table 3. In Table 4, we consider the effect of changes in η, γ, ω, τ and β/ζ. With respect to η, we consider the range interval $\eta \in [1.3, 2]$ to reflect the variety of values found in the literature (see above). With respect to γ, we explore values from 1.2 to 1.5. A value of 1.5 corresponds to a labor share of 66% in our model.[29] Regarding ω, and β/ζ, we consider a relative change of 15% (upward and downward) .

[29]In a wide class of models the ratio of price to marginal cost (the markup) is equal to the output elasticity with respect to a variable factor of production divided by the variables factor's share of revenue (e.g. De Loecker, Eeckhout and Unger, 2020; Hall, 1988). Since labor is the only factor in our model, the markup is simply the reciprocal of the labor share. Aghion et al. (2019*a*) use a a US labor share

Table 4—Sensitivity analysis

Robustness	**Loss in total innovation**
Panel A: Baseline (full sample)	5.66%
1. $\gamma = 1.20$	5.64%
2. $\gamma = 1.50$	5.71%
3. $\eta = 1.3$	9.20%
4. $\eta = 2$	2.88%
5. $\omega = 0.19$	5.63%
6. $\omega = 0.25$	5.69%
7. $\beta/\zeta = 1.44$	5.66%
8. $\beta/\zeta = 1.95$	5.66%
9. τ	
Percentile 25^{th} ($\tau = 0.005$)	1.04%
Percentile 75^{th} ($\tau = 0.046$)	10.46%
Panel B: Sub-sample of Exporting manufacturing firms	
10. Static estimation ($\tau = 0.064$)	15.22%
11. Using dynamic model ($\tau = 0.061$)	14.38%

Note: Baseline uses parameter values: ($\eta = 1.5$, $\gamma = 1.3$, $\tau = 0.0259$, $\beta/\zeta = 1.70$ and $\omega = 0.22$), see Table 3. In the robustness where γ, η, ω or β/ζ are changed, we keep τ as in the baseline. Line 9 reports the 25^{th} and 75^{th} percentile for the loss of innovation in a sample computed from 100,000 independent draws of τ from two normal distribution. The corresponding value of τ and β/ζ are computed as an average for each percentile. Lines 10-11 report the loss in total innovation when the sample is restricted to exporting manufacturing firms and Line 11 assumes a value of τ as computed using the alternative calibration presented in Section IV.B

Given that τ has been calculated using estimates of the slopes of the cross-sectional innovation-size relationship, we use our estimates of β_1 and β_2 to derive confidence intervals for τ. Specifically, we draw 100,000 values of β_1 and β_2 from two independent normal distribution $\mathcal{N}(\hat{\beta}_1, \hat{\sigma_1})$ and $\mathcal{N}(\hat{\beta}_2, \hat{\sigma_2})$, where $\hat{\beta}_i$ and $\hat{\sigma_i}$ respectively designate the point estimates and corresponding standard errors. For each of these 100,000 draws, we compute a value for τ and infer the loss in total innovation by running the model.

The results from this exercise can be found in Panel A of Table 4. As we would expect, the most important parameter is the regulatory tax, τ. From the values of β_1 and β_2, the loss is 10.5m% for the 75^{th} percentile of the distribution and 1.2% for at the 25^{th} percentile. Interestingly, η also matters: as the parameter moves from 1.3 to 2, the aggregate innovation losses falls from 9.2% to 2.9%. This is because changing η determines the elasticity of innovation with respect to R&D: as η increases, the impact of R&D on innovation decreases. Since the impact of

of GDP of 77% to obtain $\gamma = 1.3$. The French labor share after 1995 is more like 65% (see e.g. Cette, Koehl and Philippon, 2019), suggesting $\gamma = 1.5$. These values encompass most of the other estimates of the aggregate markup using other methods.

the tax comes from reducing the incentive to do R&D to grow, if R&D has little effect on growth there will be little impact of the tax. Hence, increasing η makes total innovation less sensitive to changes in τ.

By contrast, the loss in total innovation is only modestly affected by changes in γ, ω and β/ζ. This is because the tax elasticity of z only depends upon η, not on ω, γ or β/ζ. From equation (4), we see that the elasticity of innovation with respect to the regulatory tax is $1/(\eta-1)$ for large firms. Hence, changing the values of ω, γ and β/ζ only affects total innovation loss through their effects on the firm size distribution and on entry, which we know from the previous subsection plays a relatively minor quantitative role.[30]

Alternative calibration using the dynamic econometric analysis to estimate the implicit regulatory tax

Given the importance of the implicit tax for the overall impact of the regulation, we also considered estimating τ using the dynamic moments from the responsiveness to shocks rather than the static moments of the innovation-size relationship. An advantage of this approach is that it uses a better identified estimate using exogenous variation. A disadvantage is that whereas the static moment is across the whole economy, this dynamic moment is solely from the sub-sample of manufacturing firms who export (where we could construct the exogenous shifter). Re-estimating the regulatory tax in this sub-sample using the static method from our baseline in row 1 of Table 4 implies a value of $\tau = 0.064$ which is associated with a 15.2% fall in innovation (see row 10 of Panel B of Table 4). This is much larger than in the whole economy because trading manufacturing firms have a much higher level of innovation, so the cost of the regulation will be much more important.

The dynamic estimation of τ relies on the fact that after a shock ε, innovation of a firm of size $n \neq \bar{n}-1$ will be:

$$\Delta Z(n,\varepsilon) = \left(\frac{\beta\pi(n)}{\zeta\eta}\right)^{\frac{1}{\eta-1}} \omega\gamma l(n)\left((1+\varepsilon)^{\frac{1}{\eta-1}}-1\right),$$

This implies that we can calculate the cross partial of the demand shock for firms of size $n < \bar{n}$ as:

$$\frac{\partial^2 \Delta Z(n,\varepsilon)}{\partial\varepsilon\partial l} \propto (1+\varepsilon)^{\frac{2-\eta}{\eta-1}}\frac{1}{\eta-1}$$

[30]For example, as already noted a higher ω reduces the relative numbers of large firms. Since there are more firms just to the left of the regulatory threshold (whose innovation is most affected by the regulation), this makes the marginal impact of the tax slightly larger.

Similarly the cross partial for for firms of size $n \geq \bar{n}$ is:

$$\frac{\partial^2 \Delta Z(n,\varepsilon)}{\partial\varepsilon\partial l} \propto (1+\varepsilon)^{\frac{2-\eta}{\eta-1}} \frac{1}{\eta-1}(1-\tau)^{\frac{1}{\eta-1}}$$

We estimate the value of $\frac{\partial^2 \Delta Z(n,\varepsilon)}{\partial\varepsilon\partial l}$ using the procedure in Section III.C. Specifically, we estimate:

$$\begin{aligned} \Delta Z(n,\varepsilon)_{i,t} = {} & c_1 l_{i,t-2} + c_2[\mathbb{1}\left(l_{i,t-2} \geq \bar{l}\right) \times l_{i,t-2}] \\ & + c_3 \mathbb{1}\left(l_{i,t-2} \geq \bar{l}\right) + c_4[\mathbb{1}\left(l_{i,t-2} \geq \bar{l}\right) \times l_{i,t-2} \times \Delta S_{i,t-2}] \\ & + c_5[\mathbb{1}\left(l_{i,t-2} < \bar{l}\right) \times l_{i,t-2} \times \Delta S_{i,t-2}] + c_6 \Delta S_{i,t-2} + \epsilon_{i,t}. \end{aligned} \tag{11}$$

where $\mathbb{1}(l_{i,t-2} \geq \bar{l})$ is an indicator function for employment being larger than the threshold value 50. As in our baseline dynamic estimation in Section III.C, we measure $Z_{i,t}$ with log(patents), approximate the change by $\tilde{\Delta} Y_{i,t}$ and use employment and the shock at $t-2$. Details are given in Appendix D.4. Finally, we assume that ΔS is equal to the demand shock ε.[31] We then have:

$$c = \frac{c_5}{c_4} = \frac{\mathbb{E}\left[(1+\varepsilon)^{\frac{2-\eta}{\eta-1}} | l \geq \bar{l}\right](1-\tau)^{\frac{1}{\eta-1}}}{\mathbb{E}\left[(1+\varepsilon)^{\frac{2-\eta}{\eta-1}} | l < \bar{l}\right]} \tag{12}$$

where $\mathbb{E}$ is the expectations operator and is estimated using the unweighted mean from firm-year observation in the data.

We can recover τ using equation (12). Note that equation (12) is similar in form to (9) as both equations indicate how the responsiveness of large firms relative to small firms falls when the cost of regulation is higher. The expectations terms on the right hand side of equation (12) are simply adjusting the ratio to reflect the possibility that the average demand shocks hitting large firms could be different than those hitting smaller firms.

We retrieve c from c_4 and c_5 through an OLS estimation of equation (11). We add sector-year fixed effects and remove observations corresponding to firms that have a value of $l_{i,t-2}$ between 45 and 49 (as we did in the static version). We replace the expectations by their empirical counterparts and use the value of η from Table 3. This yields a value of $\tau = 0.061$ shown in row 11 of Table 4. This is extremely close to the static estimation of τ on the same sample shown in the previous row (0.064). Again, this implies a large decline of innovation in this subsector, but confirms a very similar estimate whether we use a static or dynamic

[31] We also considered an alternative approach using the fact that the theoretical elasticity of a demand shock to employment is 1. Consequently, the coefficient of a regression of Δl on ΔS, gives the link between ε and ΔS. In practice, this made no material difference to our estimate of τ.

moment.

In Appendix D.4, we also discuss several alternative dynamic estimations of τ. For example, we look at (1) restricting the sample to firms that are closer to the threshold to have a set of more comparable observations and (2) use the observations in the innovation valley and their theoretical innovation response to a shock to infer a value of τ. In all our cases, we estimate very similar estimates of the regulatory implicit tax to our dynamic baseline.

C. Welfare

Innovation increases growth which is a benefit to welfare, but it must also be paid for by diverting current consumption into R&D investments. In Schumpeterian growth models, the impact of a reduction in innovation on welfare is theoretically ambiguous. Although positive knowledge externalities generate the traditional under-investment in R&D, the business stealing effect can generate too much investment. Which dominates in our setting? Using the utility of the representative agent in equation (1), C_t is determined by the final good market clearing condition which states that each unit of final good that is produced should be used either for consumption C_t or $R\&D$. Recall that to produce an innovation intensity of $Z = nz$, a firm must spend $\zeta n z^\eta$ units of final good. We therefore have the following identity:

$$Y_t = C_t + \sum_{i \geq 1} \zeta \mu(i) i z(i)^\eta Y_t,$$

i.e. we take away R&D expenditures (there are $\mu(i)$ firms of size i) from the final good Y_t, and the residual is consumed. Denoting aggregate R&D R and plugging this into the utility function yields:

$$U = \sum_{t > 0} \beta^t \log \left[Y_0 (1+g)^t (1-R) \right] \quad \text{where} \quad R \equiv \sum_{i \geq 1} \zeta \mu(i) i z(i)^\eta$$

which can be rewritten:

$$U = \frac{\log(Y_0)}{1-\beta} + \frac{\log(1+g)\beta}{(1-\beta)^2} + \frac{\log(1-R)}{1-\beta}.$$

Since growth is defined by

$$g = \left(z_e + \sum_{i \geq 1} i z(i) \mu(i) \right) \log(\gamma),$$

and using the definition of R, we can compute total utility for any value of Y_0 using vectors z and μ and the value of z_e.

We define $g(\tau)$, $R(\tau)$ and $Y_0(\tau)$ the values of g, R and Y_0 in an economy with a regulation level equal to τ. Let $\Delta U \equiv U(\tau) - U(0)$, so

$$\Delta U = \log\left(\frac{1+g(\tau)}{1+g(0)}\right)\frac{\beta}{(1-\beta)^2} + \log\left(\frac{1-R(\tau)}{1-R(0)}\right)\frac{1}{1-\beta} + \log\left(\frac{Y_0(\tau)}{Y_0(0)}\right)\frac{1}{1-\beta},$$

denotes the difference in utility between an economy with regulation τ and an economy without regulation at the steady-state. The corresponding difference in terms of consumption equivalent is given by $\exp\left((1-\beta)\Delta U\right)$. Initial production Y_0 is equal to initial quality times the amount of labor used in production. In our baseline model, the whole labor force is employed in production with and without the regulation, as R&D does not require labor.[32] Hence, abstracting from initial quality, the effect of the regulation on welfare is governed by the first two terms in the above equation.

The first term is negative since $g(\tau) < g(0)$ due to lower innovation, hence a welfare loss from introducing the regulation. The second term is positive ($R(\tau) < R(0)$): the corresponding welfare gain stems from the fact that spending less on R&D leaves more output for consumption. The third term, although complex to quantify without stronger assumptions, can clearly be signed as negative as it is the static (non-innovation related) welfare loss that has been the focus of previous work. Hence if the sum of the first two 'dynamic' terms are negative, this will be a lower bound to the welfare loss from regulation.

With our parameter values from Table 3 and a standard value of $\beta = 0.96$, we can compute the difference in welfare in terms of the consumption equivalent. In our baseline regulated economy, welfare is 2.2% lower than in the unregulated economy. This must be added to the static efficiency losses which Garicano, Lelarge and Van Reenen (2016) estimated to be between 1.3% to 3.4%. Hence the dynamic losses from lower innovation approximately double the conventional static losses.

Table D4 in Appendix D.4 shows the welfare losses under the various alternative assumptions on the calibration values.[33]

D. *Summary on the Aggregate innovation effects of regulation*

The effects of regulation on aggregate innovation appear non-trivial. The losses are around 5.7% in our baseline estimates and even more in traded manufacturing. Four-fifths of the losses come from a lower amount of innovation across all affected firms, with the residual fifth accounted for by lower entry and a leftwards shift of

[32] This is no longer true if labor is used in production and in R&D (see section V.E). Then the tax regulation will affect Y_0 even controlling for initial quality as it will affect the fraction of labor used in production.

[33] Measuring welfare requires a separate estimation of β and ζ. The measure of welfare is obviously sensitive to the choice of β. Specifically, the welfare loss will increase as β is closer to 1 as agent gives more weight to future consumption and therefore care more about growth. When $\beta = 0.94$, welfare losses are 1.4% while when $\beta = 0.98$, welfare loss is 4.7% (see Table D4 in Appendix D.4).

the firm size distribution. Our baseline results find a (lower bound) fall in welfare of 2.2% from these dynamic losses, approximately doubling the conventional static losses. This conclusion is consistent with the important findings of Konig et al. (2022) who also emphasise that losses from skewing innovation incentives may be much greater than the conventional static misallocation losses.

V. The Nature of Innovation and Other Extensions

Our baseline model focuses on the impact of regulation on the *rate* of innovation. But there are various ways in which regulations may affect the nature of innovation. In subsection V.A, we consider an extension of our model which allows firms to invest simultaneously in two types of innovation: incremental or radical. After developing the theory we implement this empirically using two proxies for how radical a patent is: (i) a traditional future citations measure and (ii) a more novel machine learning algorithm based on the full text of the patent. Secondly, we also use textual patent analysis to measure automation as one response to the regulation may be to invest in labor saving innovations. Finally, we extend our analysis to allow for longer-lived owners and to consider R&D as scientists.

A. *Radical versus incremental innovation*

Although regulation seems to discourage overall innovation, it may also alter the *type* of innovation. A firm just below the threshold has a reduced incentive to innovate, but it might be that if she does innovate she will "swing for the fence" by investing in radical innovation. Minor, incremental innovations that just push the firm over the threshold will be strongly discouraged by the regulation. We now formalize this intuition and then test whether it has any relevance in the data.

Theory

In our baseline model, firms could only increase their number of product lines by one line in each period. In this extension, we assume that firms can now choose between: (i) Investing in an incremental innovation which augments the firm's size by one additional product line and (ii) Investing in more radical innovation which is more costly but augments the firm's size by $k > 1$ product lines. We now have four cases depending on the value for n:

1) $n < \bar{n} - k$ in which case the firm is never taxed in period 2.
2) $n < \bar{n}$ and $n \geq \bar{n} - k$ in which case the firm is taxed in period 2 only if it successfully innovated with a radical innovation.
3) $n = \bar{n} - 1$ in which case the firm is taxed in period 2 if it innovates, regardless of the type of innovation.

4) $n \geq \bar{n}$ in which case the firm is taxed in period 1 and 2 (except if the firm is at $\bar{n}+1$ but this will not affect the firm's decision)

The firm therefore chooses z and u so as to maximize:

$$n\pi(n) + \beta n z(n)\left((n+1)\pi(n+1) - n\pi(n)\right) + \beta n u(n)\left((n+k)\pi(n+k) - n\pi(n)\right) + \beta n x\left((n-1)\pi(n-1) - n\pi(n)\right) - n\zeta\left(z(n)+u(n)\right)^{\eta} - n\alpha u(n)^{\eta}, \tag{13}$$

where α denotes the additional cost of radical innovation. In Appendix C, we solve formally for u and z and in particular derive the ratio of radical over total innovation that will be use to calibrate this model.

The steady-state firm size distribution is computed in exactly the same way as in the baseline model, except that the flow equation needs to be adjusted to account for radical innovation:

$$n\mu(n)\left(u(n)+z(n)+x\right) = \mu(n-1)z(n-1)(n-1) + \mu(n+1)x(n+1) + \mu(n-k)(n-k)u(n-k),$$

with $u(n-k)$ implicitly set to 0 if $n < k$.

Calibration and Solving the model

The calibration in the model extension with two types of innovation can be done in a very similar way as in the baseline. For the additional parameters, we draw on the seminal work of Akcigit and Kerr (2018). Taking the first order condition implies:

$$u(n) = \left(\frac{\beta}{\alpha\eta}\left[(n+k)\pi(n+k) - (n+1)\pi(n+1)\right]\right)^{\frac{1}{\eta-1}}$$

and

$$z(n) = \left(\frac{\beta}{\zeta\eta}\left[(n+1)\pi(n+1) - n\pi(n)\right]\right)^{\frac{1}{\eta-1}} - \left(\frac{\beta}{\alpha\eta}\left[(n+k)\pi(n+k) - (n+1)\pi(n+1)\right]\right)^{\frac{1}{\eta-1}}$$

In this model, the ratio of total innovation $u(n)+z(n)$ of small firms (producing less than $\bar{n}-k$ goods) over large firms is still equal to $(1-\tau)^{\frac{1}{\eta-1}}$ (see Table C1). The calibration strategy to estimate τ remains identical in this model, and its value will be the same.

Additionally for small firms , the share of radical innovation over total innovation $u(n)/(z(n)+u(n))$ is equal to $\zeta/\alpha(k-1)$. In the data this ratio depends on our definition of a radical innovation. Our baseline approach is to proxy for radical innovation by selecting the top 10% patents in each technology in terms of future citations. This is consistent with Akcigit and Kerr (2018) who estimate the probability of "major advance" to be equal to 10.3%.[34] We continue to tar-

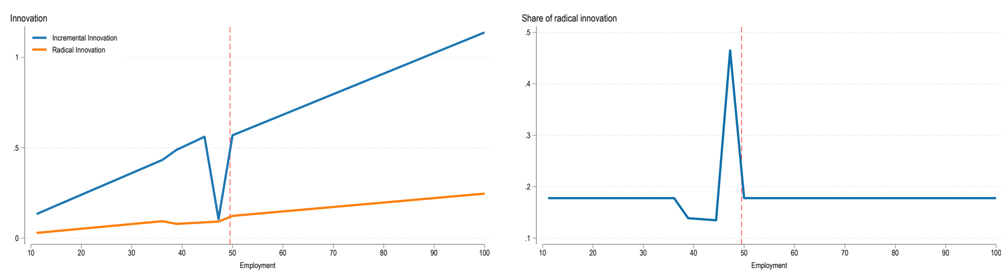
[34]This is also consistent with Acemoglu, Akcigit and Celik (2020) who also find a value between 7.8% and 13.9% (see their Table 6)

get the gap in the size distribution for ω and the long-run growth rate for β/ζ. In theory, we could estimate k using estimates of the differential step size of a radical vs incremental innovation (in our setting: γ and γ^k). Drawing again on Akcigit and Kerr (2018) finding that "External innovations that open up a new technology cluster are estimated to have more than twice the potency of internal innovations." suggests a value of k of around 4 ($3.6 = (1 + log2/log\gamma)$): a successful radical innovation corresponds to a jump of 4 lines.

We solve the model numerically using these calibration values and plot the new firm size distribution compared to the unregulated economy ($\tau = 0$) in Figure C1 (Appendix C.2). This is qualitatively similar to the model without radical innovation.

In Figure 8(a) we look at how the levels of incremental and radical innovation varies with firm employment size and also plot the share of radical innovation over total innovation in Figure 8(b). This figure suggests that the discouraging effect of regulation is substantial for incremental innovation, but close to zero for radical innovations.

Figure 7. Innovation for incremental and radical innovations



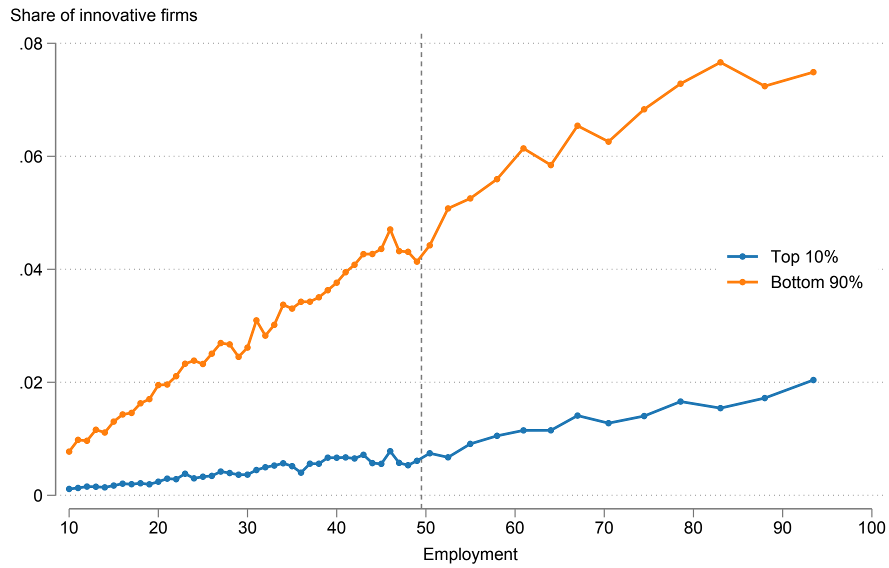
(a) Level (b) Share

Note: Left-hand side panel plots total incremental innovation $z(n)n$ (blue solid line) and total radical innovation $u(n)n$ (orange dashed line) for firms of n lines against employment in the extension where firms can choose between two types of innovations. Right-hand side panel plots the ratio of radical over total innovation $u(n)/(z(n) + u(n))$. Parameters are chosen following the calibration strategy described in Section V.A, see Table C2 in Appendix C.2.

Evidence I: Citations

We first repeat the static analysis in Figure 8 using the quality of patents as the measure of innovation output. Measuring quality using the number of future citations. For each patent within a technology class by cohort-year we determine whether the patent was in the top 10% most cited patents or in the bottom 90% (using future cites through to 2016). The two curves in Figure 8 correspond to the fractions of firms in each employment size bin respectively with patents in the top 10% cited and with patents in the bottom 90% cited. We clearly see that the

Figure 8. Share of innovative firms at each employment level and quality of innovation



Note: Share of firms with at least one priority patent in the top 10% most cited (dashed line) and the share of firms with at least one priority patent among the bottom 90% most cited in the year (solid line). All observations are pooled together. Employment bins have been aggregated so as to include at least 10,000 firms. The sample is based on all firms with initial employment between 10 and 100 (182,347 firms and 1,658,760 observations, see Panel A of Table 1) .

drop in patenting just below the regulatory threshold is barely visible for radical innovations. This is consistent with the idea that the regulation discourages low-value innovation but not higher value innovation.[35] It is also clear from the figure that the innovation-size relationship is steeper for incremental innovation than for high-value innovation. This is consistent with smaller firms accounting for a higher share of more radical innovation (e.g. Akcigit and Kerr, 2018, on US data and Manso, Balsmeier and Fleming, 2019).

Next, we repeat our preferred dynamic specification of column (5) of Table 2, but now distinguish patents of different value using their future citations. Table 5 does this for patents in the top 10%, 15% and 25% of the citation distribution in the first three columns and the patents in the complementary sets in the last three columns (i.e. the bottom 75%, 85% and 90% of the citation distribution). We clearly see that the negative effect of regulation on innovation is only statistically and economically significant for low quality patents in columns (4), (5) and (6). There are no such significant effects for patents in the top decile or quartile of the patent quality distribution (the coefficient on the interaction is even positive

[35] As for Figure 5, Figure 8 considers the innovation outcome over the whole period of observations. Variants around this can be found in Figure D3 in the Online Appendix D.

in column (2)).[36]

TABLE 5—REGRESSION RESULTS FOR DIFFERENT LEVELS OF THE QUALITY OF INNOVATION

Quality	Top 10% **(1)**	Top 15% **(2)**	Top 25% **(3)**	Bottom 75% **(4)**	Bottom 85% **(5)**	Bottom 90% **(6)**
$Shock_{t-2} \times L^{\star}_{t-2}$	-0.209	0.688	-0.826	-4.850*	-6.115**	-6.259**
	(0.847)	(0.842)	(0.936)	(2.746)	(2.635)	(2.501)
$L^{\star}_{t-2}$	-0.043	-0.019	-0.046	0.171	0.102	0.080
	(0.040)	(0.068)	(0.075)	(0.124)	(0.107)	(0.113)
$Shock_{t-2}$	-1.579	-2.287	-5.618**	-1.821	-3.884	-3.730
	(1.084)	(1.527)	(2.098)	(2.907)	(2.498)	(2.300)
$log(L)_{t-2}$	0.017	-0.010	-0.042	-0.018	-0.044	-0.060*
	(0.015)	(0.024)	(0.031)	(0.023)	(0.034)	(0.034)
$Shock_{t-2} \times log(L)_{t-2}$	0.530	0.738	1.799**	0.917	1.530*	1.492*
	(0.338)	(0.473)	(0.666)	(1.020)	(0.855)	(0.796)
Fixed Effects						
Sector×Year	✓	✓	✓	✓	✓	✓
Number Obs.	142,560	142,560	142,560	142,560	142,560	142,560

Note: Estimation results of the same model as in column 5 of Table 2. The dependent variable is the Davis and Haltiwanger (1992) growth rate in the number of priority patent applications between $t-1$ and t, restricting to the top 10% most cited in the year (column 1), the top 15% most cited in the year (column 2), the top 25% most cited in the year (column 3), the bottom 85% most cited in the year (column 4), the bottom 75% most cited in the year (column 5) and the bottom 90% most cited in the year (column 6). All models include a 2-digit NACE sector interacted with a year fixed effect and a time fixed effect interacted with the initial level of export intensity. Estimation period: 1998-2007 . Standard errors are clustered at the 2-digit NACE sector level. ***, ** and * indicate p-value below 0.01, 0.05 and 0.1 respectively.

To visualize these results, we plot the marginal effect of the demand shock on innovation by the level of firm employment in Figure 9. The blue line is the marginal effect of the shock on patents in the bottom 90% of the quality distribution based on column (6) of Table 5. Overall, the impact of the shock is positive and larger for bigger firms. However, when we approach the regulatory threshold at 50, this relationship breaks down and the marginal effect of the shock falls precipitously (and actually becomes negative). The orange line plots the marginal effect of the demand shock on high quality patents in the top decile of the citation distribution from column (1) of Table 5. This line is also positive for almost all firms and rises with firm size. By contrast, with low-value patents, there is no evidence of any sharp downturn just below the regulatory threshold.[37]

In short, there seems to be evidence that the chilling effect of regulation on innovation is not an issue for radical innovation and is instead confined to incre-

[36]We show the diminishing effect of the shock around the threshold for many other quantiles of the patent value distribution in five percentile intervals in Figure D4. This shows a clearly declining pattern.

[37]The stronger relationship between demand growth and incremental (rather than radical) innovation is consistent with the earlier cross-sectional Figure 8 and also Manso, Balsmeier and Fleming (2019).

FIGURE 9. TOTAL MARGINAL EFFECT OF A SHOCK

Note: Marginal effect of a shock at different level of employment, based on the model in column 1 and 6 of Table 5. Marginal effect is calculated on top 10% and bottom 90% most cited patents.

mental patents, which is broadly consistent with the generalization of the model we developed for two types of R&D.

EVIDENCE II: PATENT TEXT MEASURES OF NOVELTY

We construct an alternative measure of radical innovation that is made to reflect the level of novelty of a patent using the text describing the patent (in the abstract and main body). We follow Kelly et al. (2018) who build an index of novelty by looking at how much the text of a given patent differs from the current state of knowledge in the technological classes using machine learning text-to-data techniques. This measure has been shown to capture features missed by citation-based indicators (see Bergeaud, Potiron and Raimbault, 2017 for a review). For example, using many detailed industry case studies, the novelty measure has been shown to better reflect breakthrough technologies than citations (or other originality measures).

To implement this method we exploit the work of Google Patent (GP) who recently released a quantitative description of every patent (or embedding representation see Srebrovic, 2019 for details). GP embeddings use artificial intelligence analysis of text to summarize the most important features of the patent text into a vector of 64 numbers bounded between -1 and 1. We can then calculate the "distance" between any pair of patents by simply taking the dot product between the two vectors. Full details are provided in Appendix D.5, but the basic idea is that we calculate novelty by computing the distance between a patent and a reference point from past patents in the same technological field. A more novel patent will use words that are further away from the current state of the art as indexed by the typical descriptions of patents.

We replicate all the analyses of the previous subsection on citations using this new measure in Appendix D and find broadly similar results.[38] First, in Figure D6, we show that the cross-sectional patterns show no innovation valley or a falling the innovation-size gradient at 50 employees for novel patents (in fact the gradient, if anything, is steeper after 50), whereas the usual patterns emerge for non-novel patents. Second, we replicate Table 5 and split patents between the top 10% , 15% and 25% and bottom 90% , 85% and 75% based on their novelty score. Table D6 shows that the least novel (bottom 90%) patents have a significantly lower response rate to the exogenous demand shock whereas there is a small and insignificant response of the top 10% most novel.

Calculation of aggregate effects in the two types of innovation model

The finding that the main effects of regulations are on incremental innovation would seem to imply some reduction in the magnitude of the losses. A reduced form approach is given in Appendix D.5 containing firm-level employment growth regressions (Table D5) that show how although both types of innovations have a significant and positive effect on firm growth, the effect of a radical innovation is two to three times larger than that of an incremental innovation. Since most patents are incremental, this implies that innovation might only fall by about 4.4% instead of the baseline 5.7% (see Appendix D.5).

A more rigorous approach is in Appendix C.2 that re-calibrates all parameters to the new model. The new losses in welfare and total innovation are in Figure C2 and are indeed lower than those in the baseline model. The differences are less pronounced that what the reduced form approach would predict (loss of 5.3% in total innovation and 2.1% for welfare) which is mainly because the full model takes into account that although radical innovation creates more growth, it also uses more resources.

Summary on radical versus incremental innovation

Broadly, both citation and novelty based measures of radical patents are consistent with the extension to the model to allow for endogenous types of R&D. In both the theory and the data, the main effect of the regulation is to discourage only incremental innovation. This reduces the negative impact of the regulation to some degree, but far from eliminates it as even incremental innovations have social value.

[38] Note that this is not because the two measures are almost identical: the correlation between the two measures (cites vs. novelty) is only 0.1.

B. Labor-Saving Technology

There are many ways in which firms can respond to the regulation other than by reducing the pace of innovation. In addition to cutting back employment growth, Garicano, Lelarge and Van Reenen (2016) document how firms approaching the threshold also increase over time, capital investment, outsourcing and the skill mix. These might mitigate some of the costs, but will not eliminate the regulatory tax, as these are imperfect substitutes for job growth. Yet another strategy may be to develop labor saving automation technologies, that will enable the firm to increase output with less labor inputs.

To address the challenge of determining the degree to which a patent is about automation we again use textual analysis. In particular, we draw on Mann and Püttmann (2018) who used a supervised machine learning technique to classify automation and non-automation patents. Since their work was on the USPTO which is only a subsample of our data, we train an algorithm based on their classification using the GP embedding vector discussed in the previous subsection and then extrapolate this *predicted* measure of automation for all our sample. With this measure in hand, we again replicate all the analyses of the previous subsections. Consistent with our expectations, we find that the regulation only affected non-automation patenting (full results are presented in Appendix D.5). For example, Table D7 shows that faced with a positive demand shock, firms were significantly less likely to innovate in non-automation patents (bottom quartile), but were more likely to respond with automation patents (top quartile). Finally, we draw on a measure of process innovation developed by Arora et al. (2020), which are more likely to be labor saving (see Appendix D.5 for more details). This generates similar qualitative results to automation patents.

C. Longer lived owners

In our baseline model, although firms can live forever we simplified the analytical problem by assuming the owners of firms only live for two periods. We now show that the qualitative and quantitative predictions of the model carry over to a more complex environment where owners live longer. Appendix C.3 gives the details, but our strategy is to consider extending the lifetime of the owner by one extra period, solve for the new equilibrium, examine the qualitative predictions and then re-calibrate the quantitative model to look at aggregate innovation and welfare. Finally, we show that these findings extend naturally when adding an arbitrary number of additional time periods.[39]

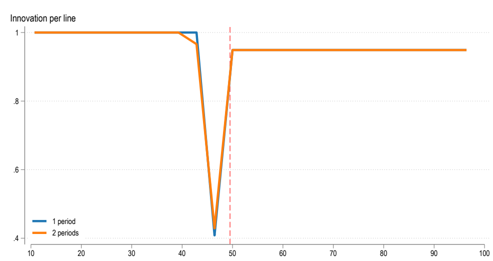
Consider extending our baseline model to allow the firm owner to live for three periods instead of the two period baseline. In the first period, the owner inherits a firm of size n_1. She then chooses her level of innovation $Z_1(n_1) = n_1 z_1(n_1)$ and

[39] In the working paper version, we show that qualitatively similar results are also found when considering another approach to modelling infinitely lived owners (Aghion, Bergeaud and Van Reenen, 2021). Unfortunately, this model does not lend itself to quantitative calibration in any straightforward manner.

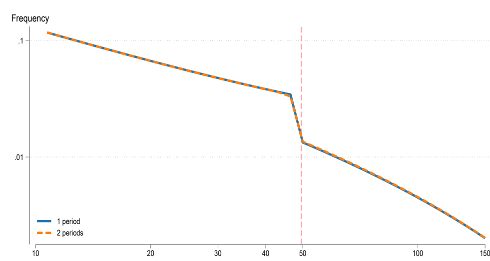
enters period 2 with a size n_2 (which can be either equal to n_1, $n_1 + 1$ or $n_1 - 1$). She chooses the level of innovation for period 2, $Z_2(n_2) = nz_2(n_2)$. Finally, the owner collects profits, exits and ownership passes on to a new agent. Because the firm's owner only produces for two periods, we refer to this model as "the two period model" while the baseline model is denoted the "one period model".

It is thus possible to solve for equilibrium innovation given the number of lines in each period. Compared to the baseline case, the regulation will not only impact firms with a size $\bar{n}-1$ but also firms with a size $\bar{n}-2$ in period 1. Figure 11(a) plots the value of $(z_1 + z_2)/2$, the average value of innovation per period, along with the value of z in the baseline model against employment. The main differences between the two is that in the extended model, the "innovation valley" is wider immediately before the 50-employee threshold, as firms anticipate the costs of being closer to the threshold at lower sizes. However, for the same reason, it makes the magnitude of the innovation drop at around 49 shallower as firms begin responding earlier in the size distribution to the threat of crossing the threshold. The fall of innovation to the right of the threshold is broadly unaltered. As the number of periods extends, the valley becomes increasingly wider and shallower (see Appendix Figure C3).

Figure 10. Innovation and firm size distribution: Comparing baseline model with longer-lived owner model



(a) Innovation



(b) Size distribution (log scale)

Note: The left-hand side panel plots total innovation per line (compared to firm employment) in our baseline model (blue solid line) compared to a model with two production periods (orange dashed line). In the latter case the average innovation over the two periods is plotted. The right-hand side panel plots the corresponding size distribution. Parameters are chosen following the calibration strategy described in Section V.C, see Table C3 in Appendix C.3.

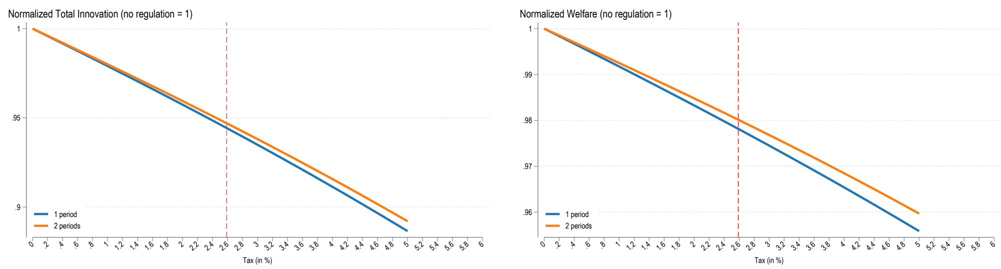
To solve for the size distribution, we look for solution where the distribution of firms in their period 2 is the same as the distribution of firms in their first period. The flow equation that determines the equilibrium size distribution is the same as in the baseline case. Figure 11(b) plots this distribution against the value of employment in the baseline case and in the two period model.

In the baseline model, the calibration of τ, which governed the aggregate innovation loss followed directly from the comparison of the slopes of the innovation

- firm size cross-sectional relationship in large vs. small firms. In our extended multi-period model, the calibration is slightly more involved and all parameters need to be estimated simultaneously. The resulting parameter values are presented in Table C3 and are very similar to those in the baseline model in Table 3.

The loss in total innovation and total welfare are shown in Figures 12(a) and 12(b) along with the corresponding loss in the baseline model. The figures show that the loss in total innovation and welfare remains very similar in the new multi-period model compared to the baseline, especially since the value of the implicit regulatory tax remains at 2.6%.

FIGURE 11. AGGREGATE INNOVATION AND WELFARE IN A MODEL WITH TWO PERIOD LIVED OWNERS



(a) Total Innovation (b) Welfare

Note: The left-hand side panel plots total innovation loss against the value of the regulation threshold τ in our baseline model (blue solid line) and in a model with two production periods (orange dashed line). The right-hand side panel plots the loss in consumption equivalent welfare. Parameters are chosen following the calibration strategy described in Appendix Section C.3, except for τ, see Table C3 in Appendix C.3.

The model can be naturally extended to adding more periods to the firm owner's life through induction and Appendix C.3 shows how the results carry over.

In summary, adding extra periods to a firm owner's life extends the "shadow" of the regulation further down the firm size distribution: the innovation valley becomes wider and flatter. A model calibration shows very similar aggregate innovation and welfare losses to our baseline case, however, suggesting that our simpler, more analytically tractable approach does not mislead us. Moreover, the theoretical findings on the shape of the innovation-size relationship generalize to having many more periods. Hence, we think our simple approach delivers losses in the right order of magnitude and would not be changed from moving to more complex dynamic models.

D. *Under-reporting of employment*

Given the cost of regulation firms have incentives to under-report employment. In Appendix A.2 we discuss these issues in detail. There is a lot of scrutiny of the employment numbers by unions, government and other agents as well as significant fines for non-compliance. This makes non-compliance costly.

An alternative dataset to FICUS is DADS, which are social security declarations. Garicano, Lelarge and Van Reenen (2016) show that the same breaks in the firm size distribution are visible in DADS as FICUS (see their Figure 4), in particular there is a clear downward shift in the power law (in log-log space) at the threshold.

The bunching of firm density at 49 employees is less visible in DADS than FICUS. Askenazy, Breda and Pecheu (2022) argue that this is because DADS is harder to manipulate than FICUS. But the blunting of the spike is particularly strong when using DADS hours data as this has much measurement error, which could cause the impression of less bunching.

Rather than viewing any employment measure from FICUS or DADS as the single "correct" one for regulatory purposes, we should regard the employment data as a signal with measurement error. Fortunately, the methods in our paper do not require to obtain the precise value of the cut-off in the empirical data. In particular, our approach utilizes differences away from the discontinuity at 50. In our baseline calibration we use the change in the gradient of the innovation-size relationship for firms in the 10 to 45 range vs. the 50-100 range to help identify the implicit tax of regulation. Similarly, in the extension where we calibrate the implicit tax using the dynamic analysis of the responses to export market size shocks, we use the data away from the threshold, again comparing responsiveness of smaller to larger firms. Hence, the identification of the aggregate costs of the regulation does not rely closely on the firms to the left of the threshold, and is therefore robust to possible under-reporting.

In summary, although there is no perfect measure of employment, the use of FICUS appears adequate for our purposes.

E. R&D as Scientists: Endogenizing Equilibrium Wages

In the baseline model, R&D is a "lab equipment" model where the equipment is bought on the world market, labor supply is fixed and the labor force is all employed as production workers. This means the labor share, ω, is constant and unaffected by the regulation. In this extension, we consider the case where R&D uses scientists as an input, which means that the labor share can change with regulation. Full details are in Appendix C.4, but we sketch the main results here.

Workers can choose to supply labor to the R&D sector or to the production sector. In this case the total employment of firm i is given by:

$$l_i = \frac{n_i}{\omega\gamma} + \zeta n_i z_i^{\eta} \equiv L(n_i, z_i),$$

where ζ is now a labor cost. Therefore l_i depends directly upon current innovation, instead of only through past innovation as reflected in its size $(n_i/(\omega\gamma))$. The employment threshold $\bar{l}$ no longer corresponds to a single number of products, but rather to a set of pairs (z, n) such that:

$$z = \frac{1}{\zeta n} \left(\bar{l} - \frac{n}{\gamma \omega} \right)^{\frac{1}{\eta}},$$

whenever $n \leq \bar{n}$.

As employment directly depends upon the level of z, so does the profit per line which is now equal to:

$$\pi(n, z) = \frac{\gamma - 1}{\gamma} \left(1 - \mathbb{1} \left[L(n, z) \geq \bar{l} \right] \tau \right)$$

The firm's problem is otherwise the same, but again the model needs to be solved numerically. Appendix C.4 shows that the qualitative effects again go through in terms of the size distribution and the firm innovation-size relationships. However, an important additional result is that the regulation reduces the equilibrium wage: the greater the tax, the greater the fall in the wage. This will mitigate the shift to the left in the size distribution.

VI. Conclusion

In this paper, we have developed a framework to analyze the impact of regulation on innovation. We applied this to France, where strong labor regulations affect firms who employ 50 or more workers. We showed both theoretically and empirically that the prospect of these regulatory costs discourages firms just below the threshold from innovating, where innovation is measured by the volume of patent applications. This relationship emerges both when looking non-parametrically at patent density around the threshold and in a parametric exercise where we examine the heterogeneous response of firms to exogenous market size shocks (from export markets). On average, firms innovate more when they experience a positive shock, but this relationship significantly weakens when a firm is just below the regulatory threshold. We then use moments from our data and the literature to calibrate the structural parameters in the model. For example, using estimates of the R&D cost function, we can back out the magnitude of the regulatory tax from the ratio between the slopes of the innovation-size relationship for large firms compared to small firms. Our baseline estimates imply an aggregate innovation (and therefore growth) loss of about 5.7% and a lower bound on the loss of welfare of about 2.2%.

This suggests larger welfare losses than existing analyses that take technology as exogenous. A caveat to this conclusion is that when we use information on citations we find that the labor regulation deters incremental innovation, but has little effect on more radical innovation. This is consistent with a generalization of the model which allows for simultaneous investment in two types of R&D, and slightly mitigates the welfare loss of the regulation.

The analysis in this paper can be extended in several directions. First, our focus in this paper was on the long-run steady state, but it is perhaps equally important to analyze the transitional dynamics triggered by policy changes, and to factor in adjustment costs. Second, the framework can be applied to many other countries and regulatory settings. Third, our analysis remained focused on the costs of the labor regulation. However, such a regulation may also bring benefits in the form of better insurance and deeper involvement of employees in the management of the firm, which in turn fosters trust between employers and employees. Future work should take such benefits into account to see if they are sufficient to overcome the costs we have identified here.

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